Test Pattern

JEE (Main)
MINOR
10-11-2024

ONLINE TEST SERIES
CONTACT PROGRAMME
(Academic Session : 2024 - 2025)

JEE(Main) : ENTHUSE + LEADER COURSE

ANSWER KEY

PART1 : PHYSICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	A	B	A	D	D	C	B	A	A	B
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	B	C	B	D	A	C	A	C	D	B
SECTION-II	Q.	1	2	3	4	5					
	A.	3	5	412	1	582					

PART 2 : CHEMISTRY

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	D	A	B	C	D	C	D	C	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	D	B	C	D	A	B	C	B	C	A
SECTION-II	Q.	1	2	3	4	5					
	A.	1	2	6	4	2					

PART 3 : MATHEMATICS

SECTION-I	Q.	1	2	3	4	5	6	7	8	9	10
	A.	C	A	C	B	B	D	C	B	D	C
	Q.	11	12	13	14	15	16	17	18	19	20
	A.	C	C	D	C	B	B	A	B	D	A
SECTION-II	Q.	1	2	3	4	5					
	A.	1	2	20	3	6					

HINT – SHEET

PART 1 : PHYSICS

SECTION-I

1. Ans (A)

I^{st} case → n = 60° of v_{max}

$v = \frac{60 \times A\omega}{100}$

n = 0.6 Aω

$\omega\sqrt{A^2 - x_1^2} = 0.6A\omega$

$A^2 - x_1^2 = 0.36A^2$

$x_1^2 = 0.64A^2$

$x_1 = 0.8A$

II^{nd} case → n = 80% of v_{max}

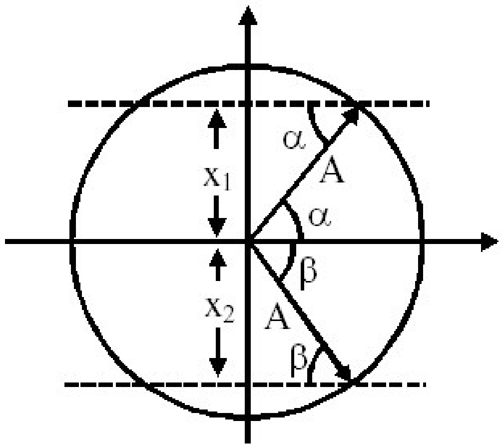
$v = \frac{80 \times A\omega}{100} = 0.8A\omega$

$\omega\sqrt{A^2 - x_2^2} = 0.8A\omega$

$\omega^2(A^2 - x_2^2) = 0.64A^2\omega^2$

$x_2^2 = 0.36A^2$

$x_2 = 0.6A$



$\sin\alpha = \frac{x_1}{A} = \frac{0.8A}{A}$

$\sin\alpha = 0.8 = \frac{4}{5} = \sin 53$

$\alpha = 53°$

$\sin\beta = \frac{x_2}{A} = \frac{0.6A}{A}$

$\sin\beta = 0.6 = \frac{3}{5} = \sin 37°$

$\beta = 37°$

$\theta = \omega t$

$(\alpha + \beta) = \omega t \Rightarrow t = \frac{90}{\omega°} = \frac{\pi \times T}{2 \times 2\pi} = \frac{T}{4}$

2. Ans (B)

$(x - A) = B\sin\omega t$

SHM of particle about x = A and its amplitude is B.

3. Ans (A)

$T = 2\pi\sqrt{\frac{M_{total}}{k_{eff}}}$

$k_{eff} = k_1 + k_2$ Parallel combination

$= 2k + k = 3k$

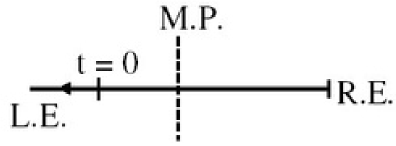
$T = 2\pi\sqrt{\frac{3m}{3k}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

4. Ans (D)

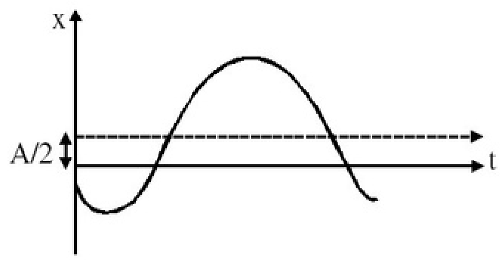
Mean position at $x = \frac{A}{2}$

at t = 0 $x = -\frac{3A}{4}$



at t = 0 it will move towards left extreme

(negative displacement)



5. Ans (D)

$a = -\beta(x - 2)$ Compares with the standard equatioh of SHM $a = -\omega^2 x$.

$\omega = \sqrt{\beta}$ (Equilibrium position x = 2)

$\therefore T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\beta}}$

∴

6. Ans (C)

The maximum K.E. of ejected photoelectron is $(K.E)_{max} = h\nu - \phi_0$

If the frequency of photon is doubled, maximum kinetic energy of photon electron becomes $(K.E.)'_{max} = 2h\nu - \phi_0$

$$\frac{(K.E.)'_{max}}{(K.E.)_{max}} = \frac{2(h\nu - \frac{\phi_0}{2})}{h\nu - \phi_0} > 2$$

Photo current $\propto \frac{\text{intensity of beam}}{h\nu}$

If intensity and frequency both are doubled, the photocurrent remains same.

7. Ans (B)

$\Delta Q = BE_p - BE_R$

$= 4\times7.2 - 4\times1.125$

$= 28.8 - 4.5 = 24.3$ MeV

8. Ans (A)

$\frac{hc}{\lambda} - \phi = \frac{1}{2}mv^2 = \frac{P^2}{2m}$

$P = \frac{h}{\lambda_d}$

$\frac{hc}{\lambda} - \phi = \frac{h^2}{2m\lambda_d^2}$

$\frac{-hc\Delta\lambda}{\lambda^2} = \frac{h^2}{2m}\frac{(-2)}{\lambda_d^3}\Delta\lambda_d$

$\frac{-hc2m}{h^2(-2)}\frac{\lambda_d^3}{\lambda^2} = \frac{\Delta\lambda_d}{\Delta\lambda}$

9. Ans (A)

$$E = \frac{13.6}{4} \times 9 = 30.6\text{eV}$$

10. Ans (B)

$E_C - E_B = \frac{hc}{\lambda_1}$ ...(1)

$E_B - E_A = \frac{hc}{\lambda_2}$ ...(2)

$E_C - E_A = \frac{hc}{\lambda_3}$ ...(3)

On add equation (1) and (2)

$E_C - E_A = hc\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right)$

$\frac{hc}{\lambda_3} = hc\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2}\right) \Rightarrow \lambda_3 = \frac{\lambda_1\lambda_2}{}$

11. Ans (B)

Let n be number of fissions per second.

Each fission produces 200 MeV.

$n\times200\times10^6$ eV is produced in one second by n fissions

But $1\text{eV} = 1.6\times10^{-19}$ J

Hence, power produce $n\times200\times10^6\times1.6\times10^{-19}$ Joule per second.

Also 1 J/s = 1 W and

1000 J/s = 1 kW.

The required power is 1 kW.

Hence, $\frac{n\times200\times10^6\times1.6\times10^{-19}}{10^3} = 1$

$n = \frac{10^3}{2\times1.6\times10^{-11}} = \frac{10^{14}}{3.2}$

$= \frac{10}{3.2}\times10^{13} = 3.125\times10^{13}$

12. Ans (C)

The range of β particle lie between zero to some maximum value. During beta decay, electrons or photons are released. Because a neutron or an antineutron is emitted simultaneously, there is an emission spectrum of electrons or positions depending on the ratio or Q reaction energies boron by the large particle. The shape of this energy curve can be predicted from the Fermi theory of beta decay.

13. Ans (B)

Spectral lines of Balmer series are in visible range.

Balmer series lines are obtained due to transition of an electron to $n = 2$ state from any higher state.

14. Ans (D)

$\lambda = \frac{h}{p}$

$\frac{\lambda_p}{\lambda_e} = \frac{p_e}{p_p} = \frac{m_e v_e}{m_p v_p}$

$2 = \frac{m_e}{m_p}\left(\frac{v_e}{4v_e}\right)$

$\therefore m_p = \frac{m_e}{8}$

15. Ans (A)

$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$ z = 2

$\frac{1}{\lambda} = 4R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

$\frac{1}{\lambda} = R\left(\frac{1}{\frac{n_1^2}{4}}-\frac{1}{\frac{n_2^2}{4}}\right)$

$\frac{n_1^2}{4}$ & $\frac{n_2^2}{4}$ must be integer & square terms

n = 4 to n = 2

16. Ans (C)

Total energy of SHM (After impulse)

$m\omega^2A^2 = \frac{1}{2}m\omega^2A_1^2$

$\therefore A_1 = \sqrt{2}A$

17. Ans (A)

$-kx = \frac{3md^2x}{dt^2}$ ... (i)

$f_3 = \frac{2md^2x}{dt^2}$ ... (ii)

18. Ans (C)

$T = 2\pi\sqrt{\frac{1}{Mgd}} = 2\pi\sqrt{\frac{2ml\alpha}{2mg\cos\frac{\alpha}{2}}}$

$= 2\pi\sqrt{\frac{1}{g\cos\left(\frac{\alpha}{2}\right)}}$

19. Ans (D)

$\lambda = \frac{h}{\sqrt{2mE}}$

$\lambda' = \frac{h}{\sqrt{2m(E-V)}}$

20. Ans (B)

Kinetic energy of ball $= \frac{h^2}{2m\lambda^2}$

Loss in kinetic energy $= \frac{h^2}{2m}\left(\frac{1}{\lambda_1^2}-\frac{1}{\lambda_2^2}\right)$

PART 1 : PHYSICS

SECTION-II

1. Ans (3)

$\lambda = \frac{h}{mv}$

velocity of an e– in H– like atom $\propto \frac{Z}{n}$

$\lambda \propto \frac{1}{v}$ so $\lambda = \frac{k}{v}$ (k = constant)

$\frac{\lambda_3}{\lambda_1} = \frac{v_1}{v_3}$

$\frac{\lambda_3}{\lambda_1} = \frac{Z_H \times n_1}{n_3 \times Z_H} = \frac{1}{3} \Rightarrow \frac{\lambda_3}{\lambda_1} = 3$

2. Ans (5)

$13.6Z^2\left[\frac{1}{4}-\frac{1}{9}\right] = 47.2$

$Z^2 \approx 25$

$Z \approx 5.00$

3. Ans (412)

$eV_S = hf - \phi$

$V_S = \frac{h}{e}f - \frac{\phi}{e}$

Slope $= \frac{h}{e} = 4.12 \times 10^{-15}$

$h = 4.12 \times 10^{-15}$ eVs

4. Ans (1)

$T = 2\pi\sqrt{\frac{\mu}{K}}$

& $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, ...$

5. Ans (582)

$y_{net} = y_1 + y_2 + y_3$

$= a[\sin\omega t + \sin(\omega t + 45°) + \sin(\omega t + 90)]$

$= (\sqrt{2}+1)\, a \sin(\omega t + 45°)$

Energy ratio is

$\frac{E_{resultant}}{E_{ring}} = \left(\frac{A}{a}\right)^2 = (\sqrt{2}+1)^2 = (3+2\sqrt{2})$

$E_{resultant} = 5.82$ J

PART 2 : CHEMISTRY

SECTION-I

1. Ans (D)

$N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$

1 3

$1-\frac{2}{}$ $^{3}3-3\times\frac{2}{5}$ $2\times\frac{2}{3}$

$KC = \frac{\left(\frac{4}{3}\right)^2}{\left(\frac{1}{3}\right)(1)} \times (\sqrt{})^{32} = 16$

$NH_3 \rightleftharpoons \frac{1}{2}N_2 + \frac{3}{2}H_2$

$KC' = \frac{1}{4}$

2. Ans (A)

$KP = KC\ (RT)^{\Delta ng}$

here $\Delta ng = 2$

3. Ans (B)

$(NH_4)_2CO_3(s) \rightleftharpoons 2NH_3(g) + CO_2(g) + H_2O(g)$

2P P P

$KP = (2P)^2.P.P = 64$ atm

P = 2 atm

Ptotal = 2P + P + P = 8 atm

4. Ans (C)

Ka x Kb = Kw

5. Ans (D)

Solubility of MX = $(4 \times 10^{-10})^{1/2}$

MX_3 : Ksp = $27s^4$

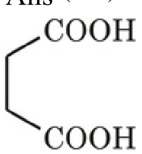
= $27\ (2 \times 10^{-5})^4$

= 4.32×10^{-18}

6. Ans (C)

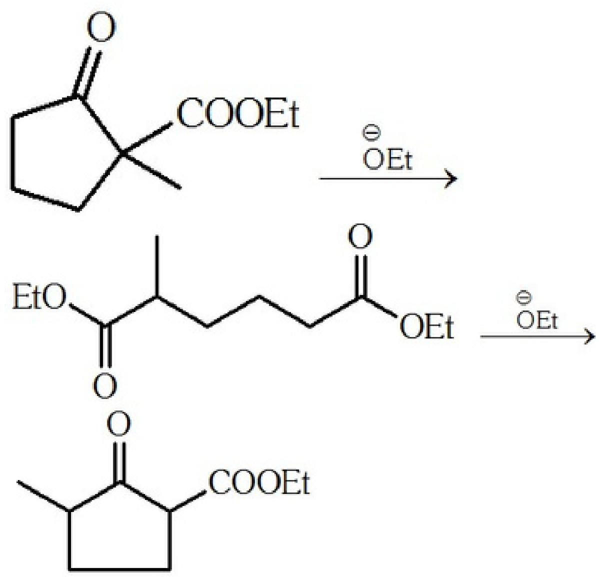
Acid halide is most reactive due to –I of Cl.

7. Ans (D)

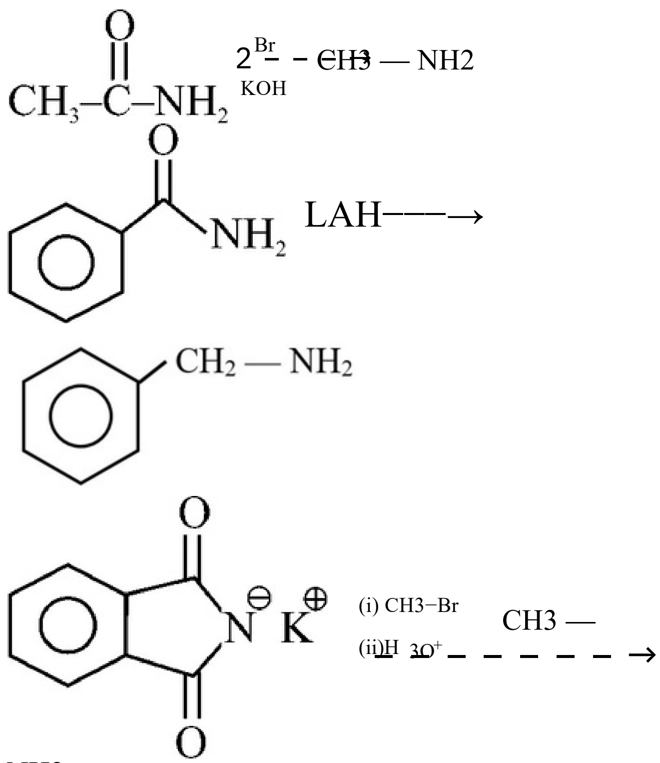


gives cyclic anhydride on heating.

8. Ans (C)



9. Ans (D)



NH2

All primary amines give isosynide test

10. Ans (C)

$CH_3CH_2OH \xrightarrow{KMnO_4} CH_3COOH\ (x)$

$\xrightarrow{SOCl_2} CH_3COCl \xrightarrow{NH_3}$

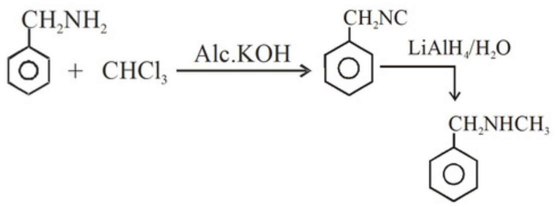
$CH_3CONH_2\ (y) \xrightarrow{Br_2/NaOH}$

$CH_3NH_2\ (z)$ (Methyl Amine)

11. Ans (D)

In azo–coupling reaction diazonium ion acts as an electrophile. The presence of NO2 group makes the diazo group more electron deficient.

12. Ans (B)



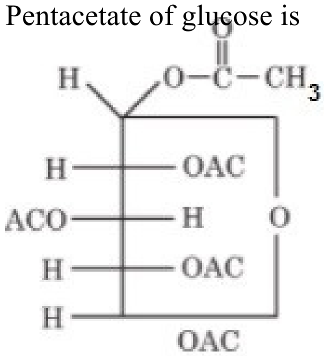
13. Ans (C)

Milk sugar (lactose) C1 - C4 β-glycosidic linkage is present.

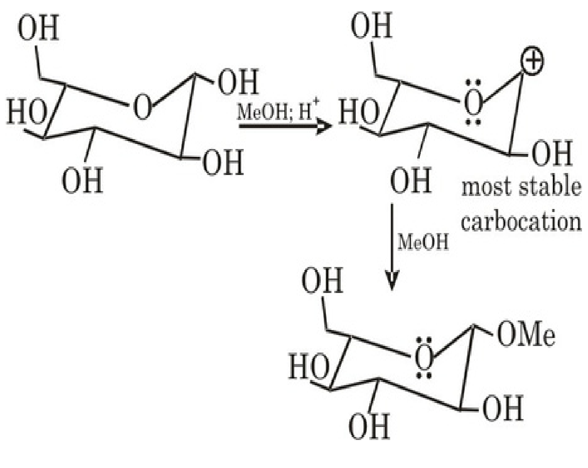
14. Ans (D)

There is no hemiactal linkage in pentacetate of glucose. So, No – CHO group in alkaline medium.

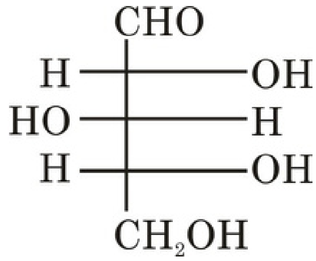
Pentacetate of glucose is



15. Ans (A)

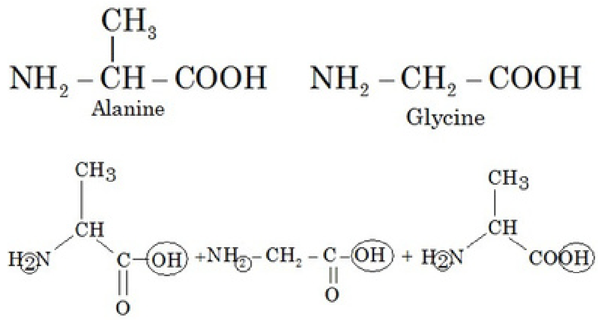


16. Ans (B)

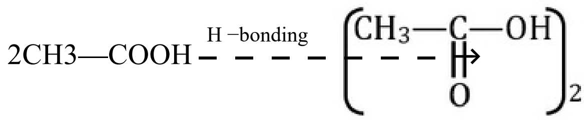


is a meso compound.

17. Ans (C)



18. Ans (B)

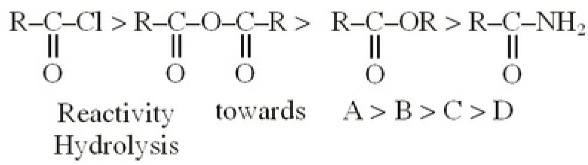


19. Ans (C)

ka α acid strength

Acid strength $\alpha \frac{-M, -H, -I}{+M, +H, +I}$

20. Ans (A)



PART 2 : CHEMISTRY

SECTION-II

1. Ans (1)

$\Delta ng = 1$

So, $\frac{KP}{KC} = (RT)1$

2. Ans (2)

$2NO + O_2 \rightarrow 2NO_2$

2 0.5 0

2 – 1 0 1

$n_i = 2.5$, $n_f = 2$

$\Delta n = 0.5$

$\Delta P = \frac{\Delta nRT}{v} = \frac{0.5 \times 0.0821 \times 300}{6.25} = 2$

3. Ans (6)

$K_h = \frac{x^2}{C} \Rightarrow x = [H^{\oplus}] = \sqrt{K_h C}$

$[H^{\oplus}] = \sqrt{10^{-9} \times 0.001} = 10^{-6}$ M

pH = 6

4. Ans (4)

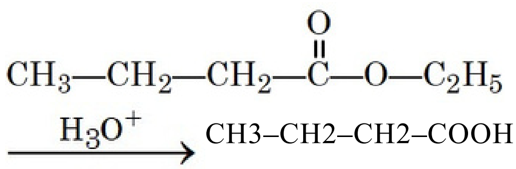
Functional group present are

–$COCH_3$ (ester) –COOH (Acid)

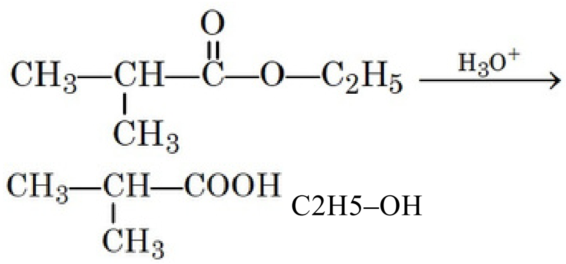
–NH_2 (Primery Amine)

–$CONH_2$ (Amide)

5. Ans (2)



$CH_3–CH_2–CH_2–COOH$ + $C_2H_5–OH$



$C_2H_5–OH$

PART 3 : MATHEMATICS

SECTION-I

1. Ans (C)

Given lines are parallel

Hence dist. between lines = diameter of circle

Hence $r = \frac{1}{2}$ (dist. between lines)

$= \frac{1}{2}\left|\frac{10-(-15)}{\sqrt{3^2+4^2}}\right| = \frac{5}{2}$

2. Ans (A)

The position vector of any point at t is

$\vec{r} = (2+t^2)\hat{i} + (4t-5)\hat{j} + (2t^2-6t)\hat{k}$

$\Rightarrow \frac{d\vec{r}}{dt} = 2t\hat{i} + 4\hat{j} + (4t-6)\hat{k}$

$\Rightarrow \left.\frac{d\vec{r}}{dt}\right|_{t=2} = 4\hat{i} + 4\hat{j} + 2\hat{k}$

and $\left|\left.\frac{d\vec{r}}{dt}\right|_{t=2}\right| = \sqrt{16+16+4} = 6$

Hence, the required unit tangent vector at t = 2 is

$\frac{1}{3}(2\hat{i} + 2\hat{j} + \hat{k})$

3. Ans (C)

Let $\alpha, \beta, \gamma \rightarrow$ angles made by direction continues with x, y, z - axis

$\alpha = \beta = \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$3\cos^2\alpha = 1$; $\cos\alpha = \pm\frac{1}{\sqrt{3}}$

p = (2, −1, 2)

Equation of line

$\frac{x-2}{\frac{1}{\sqrt{3}}} = \frac{y+1}{\frac{1}{\sqrt{3}}} = \frac{z-2}{\frac{1}{\sqrt{3}}} = k$

$Q\left(\frac{k}{\sqrt{3}}+2, \frac{k}{\sqrt{3}}-1, \frac{k}{\sqrt{3}}+2\right)$

$2\left(\frac{k}{\sqrt{3}}+2\right) + \frac{k}{\sqrt{3}} - 1 + \frac{k}{\sqrt{3}} + 2 = 9$

$\frac{4k}{\sqrt{3}} + 5 = 9$

$\frac{4k}{\sqrt{3}} = 4 \Rightarrow k = \sqrt{3}$

Point $Q \equiv (3,0,3)$

P = (2,−1,2)

$PQ = \sqrt{1+1+1} = \sqrt{3}$

4. Ans (B)

Given circles will intersect orthogonally,

if 2(1 × 0 + k × k) = 6 + k

[Using : $2(g_1g_2 + f_1f_2) = c_1 + c_2$]

$\Rightarrow 2k^2 - k - 6 = 0$

$\Rightarrow (2k + 3)(k - 2) = 0$

$\Rightarrow k = 2, -\frac{3}{2}$

5. Ans (B)

Let $\vec{a} = \lambda\vec{b} + \mu\vec{c}$

$\vec{a}$ is equally inclined to $\vec{b}$ and $\vec{d}$, where $\vec{d} = \hat{j} + 2\hat{k}$.

$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\vec{a}\cdot\vec{d}}{|\vec{a}||\vec{d}|}$

$\Rightarrow \frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{b}}{\sqrt{5}} = \frac{(\lambda\vec{b}+\mu\vec{c})\cdot\vec{d}}{\sqrt{5}}$

$\frac{[\lambda(2\hat{i}+\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})]\cdot(2\hat{i}+\hat{j})}{\sqrt{5}}$

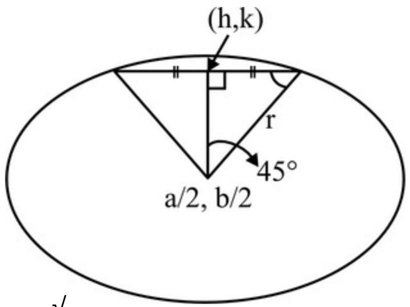
$= \frac{[\lambda(2\hat{i}+\hat{j})+\mu(\hat{i}-\hat{j}+\hat{k})]\cdot(\hat{j}+2\hat{k})}{\sqrt{5}}$

or $\lambda(4+1)+\mu(2-1) = \lambda(1)+\mu(-1+2)$

or $4\lambda = 0$, i.e., $\lambda = 0$

$\therefore \hat{a} = \frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

6. Ans (D)



$e = \sqrt{\frac{a^2}{4}+\frac{b^2}{4}} = \frac{\sqrt{a^2+b^2}}{2}$

$\cos 45^\circ = \frac{\sqrt{\left(h-\frac{a}{2}\right)^2+\left(k-\frac{b}{2}\right)^2}}{\frac{\sqrt{a^2+b^2}}{2}}$

$\frac{1}{\sqrt{2}} = \sqrt{\frac{(2x-a)^2+(2y-b)^2}{(a^2+b^2)}}$

$\frac{a^2+b^2}{2} = 4x^2+4y^2-4ax-4by+a^2+b^2$

$4x^2+4y^2-4ax-4by+\frac{a^2+b^2}{2} = 0$

$x^2+y^2-ax-by+\frac{a^2+b^2}{8} = 0$

7. Ans (C)

Given that $\vec{a}, \vec{b}$ and $\vec{c}$ are non-coplanar. Thus,

$[\vec{a}\,\vec{b}\,\vec{c}] \neq 0$ ...(i)

Again $\vec{a}\times(\vec{b}\times\vec{c})\cdot(\vec{a}\times\vec{c}) = 0$

or $[(\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}]\cdot(\vec{a}\times\vec{c}) = 0$

or $(\vec{a}\cdot\vec{c})[\vec{b}\,\vec{a}\,\vec{c}] = 0$

or $(\vec{a}\cdot\vec{c}) = 0$

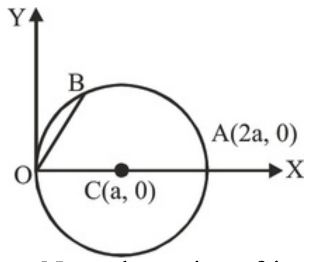
Hence, $\vec{a}$ and $\vec{c}$ are perpendicular (ii)

$\vec{a}\times(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}$

or $[\vec{a}\times(\vec{b}\times\vec{c})]\times\vec{c} = \vec{0}$.

8. Ans (B)

Here the equation of circle is $(x-a)^2+(y-0)^2 = a^2$ $\Rightarrow x^2+y^2-2ax = 0$



Now the point of intersection of circle and chord i.e.,

Put $y = mx$ in equation of circle and solve it.

O and B are O(0, 0) and B $\left(\frac{2a}{1+m^2}, \frac{2am}{1+m^2}\right)$.

Hence the equation of circle (as chord OB as diameter) is $(x^2+y^2)(1+m^2) - 2a(x+my) = 0$.

9. Ans (D)

Let $\vec{a} = \hat{i}, \vec{b} = \hat{j}, \vec{c} = \hat{k}$

$\vec{r}_1 = p\vec{a}+q\vec{b}+\vec{c} = p\hat{i}+q\hat{j}+\hat{k}$

$\vec{r}_2 = \vec{a}+p\vec{b}+q\vec{c} = \hat{i}+p\hat{j}+q\hat{k}$

$\vec{r}_1+2\vec{r}_2 = \hat{i}(p+2)+\hat{j}(q+2p)+\hat{k}(1+2q)$

$2\vec{r}_1+\vec{r}_2 = (2p+1)\hat{i}+(2q+p)\hat{j}+(2+q)\hat{k}$

$(\vec{r}_1+2\vec{r}_2)\,//\,(2\vec{r}_1+\vec{r}_2)$

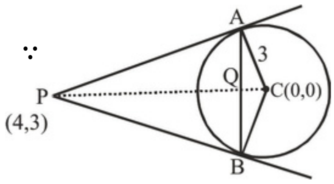
$\Rightarrow \frac{p+2}{2p+1} = \frac{q+2p}{2q+p} = \frac{1+2q}{2+q}$

$p = q$ & $q = 1$

10. Ans (C)

Equation of AB is $4x + 3y = 9$ ..... (i)

{ $\because$ it is chord of contact}



$OQ = \frac{9}{5}$ [perpendicular distance of AB from origin]

$AQ = \sqrt{OA^2 - OQ^2} = \sqrt{9 - \frac{81}{25}} = \frac{12}{5}$

$AB = 2AQ = \frac{24}{5}$

$PQ = \frac{16+9-9}{\sqrt{16+9}} = \frac{16}{5}$

Hence the area $= \frac{1}{2} \times \frac{24}{5} \times \frac{16}{5} = \frac{192}{25}$

Aliter : Required area $= \frac{a}{h^2 + k^2}(h^2 + k^2 - a^2)^{3/2}$

$= \frac{3}{4^2+3^2}(4^2 + 3^2 - 9)^{3/2} = \frac{192}{25}$

11. Ans (C)

Suppose the bisector of angle A meets BC at D. Then AD divides BC in the ratio AB : AC.

So, P.V. of D is given by

$$\frac{|\vec{AB}|(2\hat{i} + 5\hat{j} + 7\hat{k}) + |\vec{AC}|(2\hat{i} + 3\hat{j} + 4\hat{k})}{|\vec{AB}| + |\vec{AC}|}$$

But $\vec{AB} = -2\hat{i} - 4\hat{j} - 4\hat{k}$

and $\vec{AC} = -2\hat{i} - 2\hat{j} - \hat{k}$

$\Rightarrow |\vec{AB}| = 6$ and $|\vec{AC}| = 3$

Therefore, P.V. of D is given by

$$\frac{6(2\hat{i} + 5\hat{j} + 7\hat{k}) + 3(2\hat{i} + 3\hat{j} + 4\hat{k})}{6+3}$$

$= \frac{1}{3}(6\hat{i} + 13\hat{j} + 18\hat{k})$

12. Ans (C)

radius $\leqslant 5$

$\Rightarrow \sqrt{\left(\frac{\lambda}{2}\right)^2 + \left(\frac{1-\lambda}{2}\right)^2 - 5} \leqslant 5$

$\Rightarrow 2\lambda^2 - 2\lambda - 119 \leq 0$

$\Rightarrow \frac{1-\sqrt{239}}{2} \leq \lambda \leq \frac{1+\sqrt{239}}{2}$

$\Rightarrow -7.2 \leq \lambda \leq 8.2$ (nearly)

$\Rightarrow \lambda = -7, -6, ... 8$

13. Ans (D)

Here, the required plane is

$a(x - 4) + b(y - 3) + c(z - 2) = 0$

Also $a + b + 2c = 0$ and $a - 4b + 5c = 0$

Solving, we have

$\frac{a}{5+8} = \frac{b}{2-5} = \frac{c}{-4-1} = k$

$\Rightarrow \frac{a}{13} = \frac{b}{-3} = \frac{c}{-5} = k$

Therefore, the required equation of plane is

$-13x + 3y + 5z + 33 = 0$

14. Ans (C)

$\vec{r} = (1+\lambda-\mu)\hat{i} + (2-\lambda)\hat{j} + (3-2\lambda+2\mu)\hat{k}$

$\Rightarrow \vec{r} = (\hat{i}+2\hat{j}+3\hat{k}) + \lambda(\hat{i}-\hat{j}-2\hat{k}) + \mu(-\hat{i}+2\hat{k})$

which is a plane passing through $\vec{a} = \hat{i}+2\hat{j}+3\hat{k}$ and parallel to the vectors $\vec{b} = \hat{i}-\hat{j}-2\hat{k}$ and $\vec{c} = -\hat{i}+2\hat{k}$ therefore, it perpendicular to the vector

$\vec{n} = \vec{b} \times \vec{c} = -2\hat{i} - \hat{k}$

Hence, its vector equation is

$(\vec{r} - \vec{a}).\vec{n} = 0$

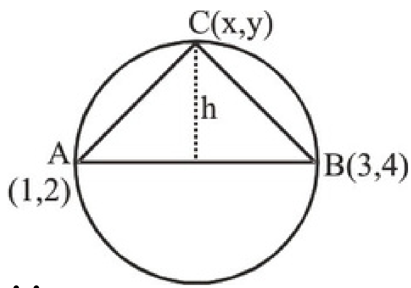
$\Rightarrow \vec{r} \cdot \vec{n} = \vec{a} \cdot \vec{n}$

$\Rightarrow \vec{r} \cdot (-2\hat{i} - \hat{k}) = -2 - 3$

$\Rightarrow \vec{r} \cdot (2\hat{i} + \hat{k}) = 5$

$\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (2\hat{i} + \hat{k}) = 5$ or $2x + z = 5$

15. Ans (B)



$\because$ A & B are end's of diameter, diameter $= 2\sqrt{2}$

$\therefore$ radius $= \sqrt{2}$

Let height of ΔABC is h.

Now, $\frac{1}{2}$.Base $\times$ h = 1 {Base=daimeter}

$\frac{1}{2} \times 2\sqrt{2} \times h = 1$

$\Rightarrow h = \frac{1}{\sqrt{2}}$

$\Rightarrow h < r$

$\therefore$ therefore no. of position of C is 4.

16. Ans (B)

Let the components of the line segment vector be a, b, c then

$a^2 + b^2 + c^2 = (63)^2$ ...(i)

also $\frac{a}{3} = \frac{b}{-2} = \frac{c}{6} = \lambda$ (say)

$\Rightarrow a = 3\lambda, b = -2\lambda, c = 6\lambda$

$\Rightarrow 49\lambda^2 = (63)^2$

$\Rightarrow \lambda = \pm \frac{63}{7} = \pm 9$

$\because a = 3\lambda < 0$

$\because$ line makes abtuse angle with x-axis

$\Rightarrow \lambda = -9$

17. Ans (A)

Let the equation be $x^2 + y^2 + 2gx + 2fy + c = 0$

it passes through (–1, –3) and (3, 0) therefore

$10 - 2g - 6f + c = 0$ ....(i)

$9 + 6g + c = 0$ ....(ii)

Slope of tangent = –4/3

$\left(\frac{0 + f}{3 + g}\right)\left(\frac{-4}{3}\right) = -1$

$\Rightarrow 3g - 4f + 9 = 0$ ....(iii)

18. Ans (B)

Let the equation of line passes through

$\frac{x-1}{a} = \frac{y-0}{b} = \frac{z+1}{c}$ ..(1)

It is pependicula to two given lines

$\therefore 2a + 7b - 3c = 0$ ..(2)

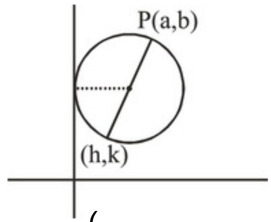
from (2) & (3)

$\frac{a}{29} = \frac{b}{-16} = \frac{c}{-18}$

$\therefore$ Equation of line

$\frac{x-1}{29} = \frac{y-0}{-16} = \frac{z+1}{-18}$

19. Ans (D)



centre $\left(\frac{h+a}{2}, \frac{k+b}{2}\right)$

radius = $\frac{1}{2}\sqrt{(h-a)^2 + (k-b)^2}$

Acc. to condition

$\frac{1}{2}\sqrt{(h-a)^2+(k-b)^2} = \frac{h+a}{2}$

$(k - b)^2 = 4ah$

$\Rightarrow (y - b)^2 = 4ax$

20. Ans (A)

Let planeis $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ ..... (i)

mid point of P(1, 2, 3) and Q(–3, 4, 5)

i.e., –1, 3, 4 lie on Eq. (i)

$\therefore \frac{-1}{a} + \frac{3}{b} + \frac{4}{c} = 1$ ..... (ii)

Also, PQ is parallel to normal of the plane (i)

$\frac{1/a}{-4} = \frac{1/b}{2} = \frac{1/c}{2}$

$\frac{1}{-2a} = \frac{1}{b} = \frac{1}{c} = \lambda$(say)

$\Rightarrow \frac{1}{a} = -2\lambda, \frac{1}{b} = \lambda, \frac{1}{c} = \lambda$

$\therefore$ From Eq. (ii), $2\lambda + 3\lambda + 4\lambda = 1$

$\therefore \lambda = \frac{1}{9}$

$a = -\frac{1}{2\lambda}$, $b = \frac{1}{\lambda}$, $c = \frac{1}{\lambda}$

$a = -\frac{9}{2}, b = 9, c = 9$

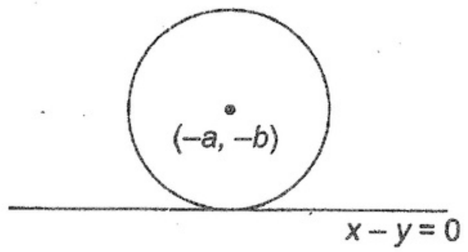
Intercepts are $\left(-\frac{9}{2}, 9, 9\right)$.

PART 3 : MATHEMATICS

SECTION-II

1. Ans (1)

Radical axis, $x - y = 0$



Centre of 1st circle is $(-a, -b)$ and $r=\sqrt{a^2+b^2-c}$.

Now perpendicular distance from center on radical axis = radius of the circle

$(a - b)^2 = 2[a^2 + b^2 - c]$

$\Rightarrow a^2 + b^2 - 2ab = 2[a^2 + b^2 - c]$

$\frac{(a + b)^2}{2c} = 1$

2. $\Rightarrow$ Ans (2)

Given, $\frac{(\vec{a}\cdot\vec{b})\vec{a}}{|\vec{a}|^2} = \frac{4}{3}(\hat{i}-\hat{j}-\hat{k})$

$\frac{\{(\lambda\hat{i}-3\hat{j}+\hat{k})\cdot(\hat{i}-\hat{j}-\hat{k})\}(\hat{i}-\hat{j}-\hat{k})}{(1+1+1)}$

$\Rightarrow (\hat{i}-\hat{j}-\hat{k}) = \frac{4}{3}$

$(\lambda + 3 - 1)(\hat{i}-\hat{j}-\hat{k}) = 4(\hat{i}-\hat{j}-\hat{k})$

$\Rightarrow (\lambda + 2)(\hat{i}-\hat{j}-\hat{k}) = 4(\hat{i}-\hat{j}-\hat{k})$

On equating the coefficient of $\hat{i}$, we get

$\lambda + 2 = 4 \quad \lambda = 2$

$\Rightarrow$

3. Ans (20)

Since, the given line touches the given circle, the length of the perpendicular from the centre (2, 4) of the circle to the line $3x - 4y - k = 0$ is equal to the radius $\sqrt{4+16+5}=5$ of the circle.

$\therefore \frac{3 \times 2 - 4 \times 4 - k}{\sqrt{9+16}} = \pm 5$

$\Rightarrow k = 15 \quad [\because k > 0]$

$3x - 4y - 15 = 0 \quad \ldots (1)$

Let equation of normal to circle $4x + 3y = \lambda$

It passes through centre (2, 4) $\Rightarrow \lambda = 20$

hence equation of normal is

$4x + 3y = 20 \quad \ldots (2)$

Solve (1) & (2)

$a = 5, b = 0 ; k + a + b = 15 + 5 + 0 = 20$

4. Ans (3)

$$\text{Shortest distance} = \frac{\left|\left[\bar{a}-\bar{c}\ \ \bar{b}\ \ \bar{d}\right]\right|}{\left|\bar{b}\times\bar{d}\right|}$$

Here $\bar{a} = 3\bar{i}+8\bar{j}+3\bar{k}, \bar{b} = 3\bar{i}-\bar{j}+\bar{k}, \bar{c} = -3\bar{i}-7\bar{j}+6\bar{k}, \bar{d} = -3\bar{i}+2\bar{j}+4\bar{k}$

$\therefore \text{Shortest distance} = \frac{270}{\sqrt{270}} = \sqrt{270} = 3\sqrt{30} = \lambda\sqrt{30} \Rightarrow \lambda = 3$

5. Ans (6)

Let the equation of the circle be

$(x - a)^2 + (y - a)^2 = a^2, a > 0,$

It touches $4x + 3y - 12 = 0$

$\therefore \frac{|4a+3a-12|}{5} = a$

$7a - 12 = \pm 5a$

take '+' sign	take '–' sign
$2a = 12$	$12a = 12$
$a = 6$	$a = 1$

$\therefore$ radius of larger circle $a = 6$