JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 29 TH JANUARY 2025) TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION–A

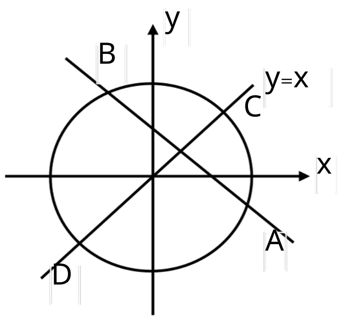
١. Let the line x + y = ١ meet the circle x٢ + y٢ = ٤ at the points A and B. If the line perpendicular to AB and passing through the mid point of the chord AB intersects the circle at C and D, then the area of the quadrilateral ADBC is equal to

(١) ٣√٧ (٢) ٢√١٤

(٣) ٥√٧ (٤) √١٤

Ans. (٢)

Sol.



By solving x = y with circle

We get

C(√٢, √٢)

D(−√٢, −√٢)

By solving x+y= ١ with circle x٢+y٢=٤

we set

$A\left(\frac{١ + √٧}{٢}, \frac{١ − √٧}{٢}\right)$

& $B\left(\frac{١ − √٧}{٢}, \frac{١ + √٧}{٢}\right)$

∴Area of Quadrilateral ACBD

= ٢ × Area of ΔBCD

$= ٢ \times \frac{١}{٢} \begin{vmatrix} \frac{١ − √٧}{٢} & \frac{1 + √٧}{٢} & ١ \\ √٢ & √٢ & ١ \\ −√٢ & −√٢ & ١ \end{vmatrix}$

= ٢√١٤

٢. Let M and m respectively be the maximum and the minimum values of

$f(x) = \begin{vmatrix} ١ + \sin^٢ x & \cos^٢ x & ٤\sin ٤x \\ \sin^٢ x & ١ + \cos^٢ x & ٤\sin ٤x \\ \sin^٢ x & \cos^٢ x & ١ + ٤\sin ٤x \end{vmatrix}, x \in R$

Then M٤ − m٤ is equal to :

(١) ١٢٨٠ (٢) ١٢٩٥

(٣) ١٠٤٠ (٤) ١٢١٥

Ans. (١)

Sol. $\begin{vmatrix} ١ + \sin^٢ x & \cos^٢ x & ٤\sin ٤x \\ \sin^٢ x & ١ + \cos^٢ x & ٤\sin ٤x \\ \sin^٢ x & \cos^٢ x & ١ + ٤\sin ٤x \end{vmatrix}, x \in R$

R٢ → R٢ − R١ & R٣ → R٣ − R١

$f(x) = \begin{vmatrix} ١ + \sin^٢ x & \cos^٢ x & ٤\sin ٤x \\ −١ & ١ & ٠ \\ −١ & ٠ & ١ \end{vmatrix}$

Expand about R١, we get

f(x) = ٢ + ٤sin٤x

∴M = max value of f(x) = ٦

m = min value of f(x) = −٢

∴M٤ − m٤ = ١٢٨٠

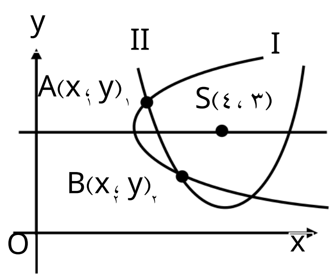
٣. Two parabolas have the same focus (٤, ٣) and their directrices are the x-axis and the y-axis, respectively. If these parabolas intersects at the points A and B, then (AB)٢ is equal to

(١) ١٩٢ (٢) ٣٨٤

(٣) ٩٦ (٤) ٣٩٢

Ans. (١)

Sol.



Let intersection points of these two parabolas are

$A(x_١, y_١)$ & $B(x_٢, y_٢)$

□□□ equation of parabola I and II are given below

$□(x-٤)^٢+(y-٣)^٢ = x^٢$ ....(١)

& $(x-٤)^٢+(y-٣)^٢ = y^٢$ ....(٢)

Here $A(x_١, y_١)$ & $B(x_٢, y_٢)$ will satisfy the equation

Also from equations (١) & (٢), we get $x = y$ ..(٣)

Put $x = y$ in equation (١)

We get $x^٢ - ١٤x + ٢٥ = ٠$

$x_١ + x_٢ = ١٤$

$x_١x_٢ = ٢٥$

$□AB = \sqrt{(x_١-x_٢)^٢+(y_١-y_٢)^٢}$

$= \sqrt{٢(x_١-x_٢)^٢}$

$= \sqrt{٢[(x_١+x_٢)^٢-٤x_١x_٢]}$

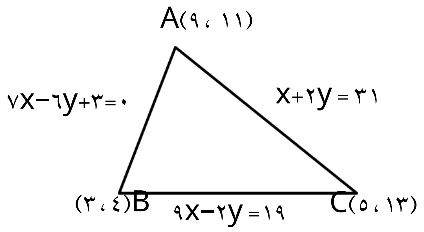
$= ١٩٢$

٤. Let ABC be a triangle formed by the lines $٧x-٦y+٣=٠$, $x+٢y-٣١=٠$ and $٩x-٢y-١٩=٠$. Let the point (h, k) be the image of the centroid of □ABC in the line $٣x+٦y-٥٣=٠$. Then $h^٢+k^٢+hk$ is equal to

(١) ٣٧ (٢) ٤٧
(٣) ٤٠ (٤) ٣٦

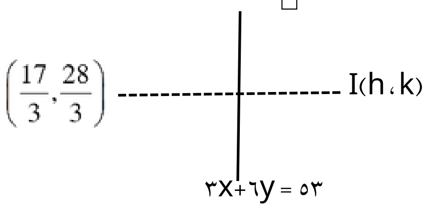
Ans. (١)

Sol.



□ centroid of □ABC $= \left(\frac{٩+٣+٥}{٣}, \frac{١١+٤+١٣}{٣}\right)$

$= \left(\frac{١٧}{٣}, \frac{٢٨}{3}\right)$



Let image of centroid with respect to line mirror is (h, k)

$\Rightarrow \left(k - \frac{٢٨}{٣}\right) \Big/ \left(h - \frac{١٧}{٣}\right) = ٢$ □

$\frac{h + \frac{١٧}{٣}}{٢}$ & $٣\left(\frac{h + \frac{١٧}{٣}}{٢}\right) + ٦\left(\frac{k + \frac{٢٨}{٣}}{٢}\right) = ٥٣$

Solving (١) & (٢) we get $h = ٣, k = ٤$

$□h^٢ + k^٢ + hk = ٣٧$

٥. Let $a = ٢\hat{i} - \hat{j} + ٣\hat{k}$, $b = ٣\hat{i} - ٥\hat{j} + \hat{k}$ and c be a vector such that $a \times c = c \times b$ and $(a + c) \cdot (b + c) = ١٦٨$. Then the maximum value of $|c|^٢$ is :

(١) ٧٧ (٢) ٤٦٢
(٣) ٣٠٨ (٤) ١٥٤

Ans. (٣)

Sol. $a = ٢\hat{i} - \hat{j} + ٣\hat{k}$

$b = ٣\hat{i} - ٥\hat{j} + \hat{k}$

$a \times c = c \times b$

$a \times c + b \times c = ٠$

$(a + b) \times c = ٠$

$\Rightarrow c = \lambda(a + b)$

$c = \lambda(٥\hat{i} - ٦\hat{j} + ٤\hat{k})$ ....(١)

$|c|^٢ = \lambda^٢(٢٥ + ٣٦ + ١٦)$

$|c|^٢ = ٧٧\lambda^٢$

$(a + c) \cdot (b + c) = ١٦٨$

$a \cdot b + a \cdot c + c \cdot b + |c|^٢ = ١٦٨$

$١٤ + c \cdot (a + b) + ٧٧\lambda^٢ = ١٦٨$

using equation (١)

$\lambda|٥\hat{i} - ٦\hat{j} + ٤\hat{k}|^٢ + ٧٧\lambda^٢ = ١٥٤$

$٧٧\lambda^٢ + ٧٧\lambda - ١٥٤ = ٠$

$\lambda^٢ + \lambda - ٢ = ٠$

$\lambda = -٢, ١$

□Maximum value of $|c|^٢$ occurs when $\lambda = -٢$

$|c|^٢ = ٧٧\lambda^٢$

$= ٧٧ \times ٤$

$= ٣٠٨$

٦. Let P be the set of seven digit numbers with sum of their digits equal to ١١. If the numbers in P are formed by using the digits ١،٢ and ٣ only، then the number of elements in the set P is:

(١) ١٥٨ (٢) ١٧٣

(٣) ١٦٤ (٤) ١٦١

Ans. (٤)

Sol. (i) number of numbers created using

$١١١١١٣٣ = \frac{٧!}{٥!٢!} = ٢١$

(ii) number of numbers created using

$١١١١٢٢٣ = \frac{٧!}{٤!٢!} = ١٠٥$

(iii) number of numbers created using

$١١١٢٢٢٢ = \frac{٧!}{٤!٣!} = ٣٥$

Total = ١٦١

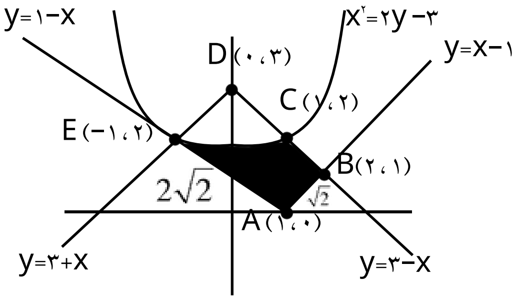
٧. Let the area of the region $\{(x, y) : ٢y \le x^٢ + ٣, y + |x| \le ٣, y \ge |x - ١|\}$ be A. Then ٦A is equal to:

(١) ١٦ (٢) ١٢

(٣) ١٨ (٤) ١٤

Ans. (٤)

Sol.



A = Rectangle ABDE − Area of region EDC

$A = ٤ - ٢\int_{٠}^{١}\left((٣ - x) - \left(\frac{x^٢ + 3}{٢}\right)\right)dx$

$A = ٤ - ٢\left[٣x - \frac{x^٢}{٢} - \frac{x^٣}{٦} - \frac{٣}{٢}x\right]_0^١$

$A = ٤ - ٢\left[3 - \frac{١}{٢} - \frac{١}{٦} - \frac{٣}{٢}\right] = \frac{٧}{٣}$

So ٦A = ١٤

٨. The least value of n for which the number of integral terms in the Binomial expansion of $\left(\sqrt[٣]{٧} + \sqrt[١٢]{١١}\right)^n$ is ١٨٣، is:

(١) ٢١٨٤ (٢) ٢١٤٨

(٣) ٢١٧٢ (٤) ٢١٩٦

Ans. (١)

Sol. General term $= {}^nC_r \left(٧^{١/٣}\right)^{n-r}\left(١١^{١/١٢}\right)^r$

$= {}^nC_r (٧)^{\frac{n-r}{٣}}(١١)^{r/١٢}$

For integral terms، r must be multiple of ١٢

$\Rightarrow r = ١٢k، k \in W$

Total values of r = ١٨٣

Hence max r = ١٢(١٨٢)

$\Rightarrow ٢١٨٤$

Min value of n = ٢١٨٤

٩. The number of solutions of the equation

$\left(\frac{٩}{x} - \frac{٩}{\sqrt{x}} + ٢\right)\left(\frac{٢}{x} - \frac{٧}{\sqrt{x}} + 3\right) = 0$ is:

(١) ٢ (٢) ٤

(٣) ١ (٤) ٣

Ans. (٢)

Sol. Consider $\frac{١}{\sqrt{x}} = t$ $\boxed{x \ne ٠}$

$(٩t^٢ - ٩t + ٢)(٢t^٢ - ٧t + ٣) = ٠$

$(٣t - ٢)(٣t - ١)(t - ٣)(٢t - ١) = ٠$

$\Rightarrow t = \frac{١}{٣}، \frac{١}{٢}، \frac{٢}{٣}، ٣$

$x = ٩، ٤، \frac{٩}{٤}، \frac{١}{٩}$

So، no. of solutions = ٤

10. Let $y = y(x)$ be the solution of the differential equation $\cos x(\log_e(\cos x))^2 dy + (\sin x - 3y\sin x \log_e(\cos x))dx = 0$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. If $y\left(\frac{\pi}{4}\right) = \frac{-1}{\log_e 2}$, then $y\left(\frac{\pi}{6}\right)$ is :

(1) $\frac{2}{\log_e(3) - \log_e(4)}$ (2) $\frac{1}{\log_e(4) - \log_e(3)}$

(3) $-\frac{1}{\log_e(4)}$ (4) $\frac{1}{\log_e(3) - \log_e(4)}$

Ans. (4)

Sol. $\cos x(\ln(\cos x))^2 dy + (\sin x - 3y(\sin x)\ln(\cos x))dx = 0$

$\cos x(\ln(\cos x))^2 \frac{dy}{dx} + \sin x = 3\sin x \cdot \ln(\cos x)\, y$

$\frac{dy}{dx} - \frac{3\tan x}{\ln(\cos x)} y = \frac{-\tan x}{(\ln(\cos x))^2}$

$\frac{dy}{dx} + \frac{3\tan x}{\ln(\sec x)} y = \frac{-\tan x}{(\ln(\sec x))^2}$

$\text{I.F.} = e^{\int \frac{3\tan x}{\ln(\sec x)}dx} = (\ln(\sec x))^3$

$y(\ln(\sec x))^3 = \int \frac{-\tan x}{(\ln(\sec x))^2}(\ln(\sec x))^3 dx$

$y(\ln(\sec x))^3 = -\frac{(\ln(\sec x))^2}{2} + C$

Given : $x = \frac{\pi}{4}$, $y = \frac{-1}{\ln 2}$

$\frac{-1}{\ln 2}(\ln\sqrt{2})^3 = -\frac{1}{2}(\ln\sqrt{2})^2 + C$

$\Rightarrow \frac{-1}{8\ln 2}(\ln 2)^3 = -\frac{1}{2}\cdot\frac{1}{4}(\ln 2)^2 + C$

$\Rightarrow -\frac{1}{8}(\ln 2)^2 = -\frac{1}{8}(\ln 2)^2 + C$

$\Rightarrow C = 0$

$\therefore y(\ln(\sec x))^3 = -\frac{1}{2}(\ln(\sec x))^2 + 0$

$y = \frac{-1}{2\ln(\sec x)}$

$y = \frac{1}{2\ln(\cos x)}$

$\therefore y\left(\frac{\pi}{6}\right) = \frac{1}{2\ln\left(\cos\frac{\pi}{6}\right)}$

$= \frac{1}{2\ln\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{1}{2\left(\frac{1}{2}\ln 3 - \ln 2\right)}$

$= \frac{1}{\ln 3 - \ln 4}$

Option (4)

11. Define a relation R on the interval $\left[0, \frac{\pi}{2}\right)$ by x R y if and only if $\sec^2 x - \tan^2 y = 1$. Then R is :

(1) an equivalence relation

(2) both reflexive and transitive but not symmetric

(3) both reflexive and symmetric but not transitive

(4) reflexive but neither symmetric not transitive

Ans. (1)

Sol. $\sec^2 x - \tan^2 x = 1$ (on replacing y with x)

$\Rightarrow$ Reflexive

$\sec^2 x - \tan^2 y = 1$

$\Rightarrow 1 + \tan^2 x + 1 - \sec^2 y = 1$

$\Rightarrow \sec^2 y - \tan^2 x = 1$

$\Rightarrow$ symmetric

$\sec^2 x - \tan^2 y = 1,$

$\sec^2 y - \tan^2 z = 1$

Adding both

$\Rightarrow \sec^2 x - \tan^2 y + \sec^2 y - \tan^2 z = 1 + 1$

$\sec^2 x + 1 - \tan^2 z = 2$

$\sec^2 x - \tan^2 z = 1$

$\Rightarrow$ Transitive

hence equivalence relation

Option (1)

١٢. Let the ellipse, $E_1 : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, a < b and $E_2 : \frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$, A > B have same eccentricity $\frac{1}{\sqrt{3}}$. Let the product of their lengths of latus rectums be $\frac{32}{\sqrt{3}}$, and the distance between the foci of E_1 be ٤. If E_1 and E_2 meet at A, B, C and D, then the area of the quadrilateral ABCD equals:

(١) ٦ $\sqrt{6}$ (٢) $\frac{18\sqrt{6}}{5}$

(٣) $\frac{12\sqrt{6}}{5}$ (٤) $\frac{24\sqrt{6}}{5}$

Ans. (٤)

Sol. $2ae = 4$

$2a\left(\frac{1}{\sqrt{3}}\right) = 4$

$\Rightarrow a = 2\sqrt{3}$

$1 - \frac{b^2}{12} = \frac{1}{3} \Rightarrow b^2 = 8$

Now $\frac{2A^2}{B} \cdot \frac{2b^2}{a} = \frac{32}{\sqrt{3}} \Rightarrow \frac{2 \times 8}{2\sqrt{3}} \cdot \frac{2A^2}{B} = \frac{32}{\sqrt{3}}$

$\Rightarrow A^2 = 2B$

$\Rightarrow 1 - \frac{B^2}{A^2} = \frac{1}{3} \Rightarrow 1 - \frac{B^2}{2B} = \frac{1}{3} \Rightarrow B = 3$

$\Rightarrow A^2 = 6$

$\Rightarrow \frac{x^2}{12} + \frac{y^2}{8} = 1$ .....(1)

$\frac{x^2}{6} + \frac{y^2}{9} = 1$ .....(2)

On solving (1) & (2) we get

$(x, y) = \left(\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(-\frac{\sqrt{6}}{\sqrt{5}}, \frac{6}{\sqrt{5}}\right), \left(\frac{\sqrt{6}}{\sqrt{5}}, -\frac{6}{\sqrt{5}}\right), \left(-\frac{\sqrt{6}}{\sqrt{5}}, -\frac{6}{\sqrt{5}}\right)$

The four points are vertices of rectangle and its area = $\frac{24\sqrt{6}}{5}$

١٣. Consider an A.P. of positive integers, whose sum of the first three terms is ٥٤ and the sum of the first twenty terms lies between ١٦٠٠ and ١٨٠٠. Then its ١١th term is:

(١) ٨٤ (٢) ١٢٢

(٣) ٩٠ (٤) ١٠٨

Ans. (٣)

Sol. $S_3 = 3a + 3d = 54$

$\Rightarrow a + d = 18$

$S_{20} = 10(2a + 19d)$

$= 10(36 + 17d)$

$\Rightarrow 1600 < 10(36 + 17d) < 1800$

$\Rightarrow 160 < 36 + 17d < 180$

$\Rightarrow 124 < 17d < 144$

$\Rightarrow 7\frac{5}{17} < d < 8\frac{8}{17}$

Common difference will be natural number

$\Rightarrow d = 8 \Rightarrow a = 10$

$\Rightarrow a_{11} = 10 + 10 \times 8 = 90$

١٤. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 7\hat{j} + 3\hat{k}$. Let

$L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda\vec{a}, \lambda \in R$ and

$L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu\vec{b}, \mu \in R$ be two

line L_3 passes through the point of intersection of L_1 and L_2, and is parallel to $\vec{a} + \vec{b}$, then L_3 passes through the point:

(١) (٨, ٢٦, ١٢) (٢) (٢, ٨, ٥)

(٣) (−١, −١, ١) (٤) (٥, ١٧, ٤)

Ans. (١)

Sol. $L_1 : \vec{r} = (-\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \vec{r} = (\lambda - 1)\hat{i} + 2(\lambda + 1)\hat{j} + (\lambda + 1)\hat{k}$

$L_2 : \vec{r} = (\hat{j} + \hat{k}) + \mu(2\hat{i} + 7\hat{j} + 3\hat{k})$

$\Rightarrow \vec{r} = 2\mu\hat{i} + (1 + 7\mu)\hat{j} + (1 + 3\mu)\hat{k}$

For point of intersection equating respective components

$\lambda - 1 = 2\mu$

$2(\lambda + 1) = 1 + 7\mu$

$\lambda + 1 = 1 + 3\mu$

$\Rightarrow$ We get

$\Rightarrow \lambda = 3$ and $\mu = 1$

$\therefore \vec{a} - \vec{b} = 3\hat{i} + 9\hat{j} + 4\hat{k}$

$L_3: \vec{r} = 2\hat{i} - 8\hat{j} + 4\hat{k} + t(3\hat{i} + 9\hat{j} + 4\hat{k})$

For $t = 2$, $\vec{r} = 8\hat{i} + 10\hat{j} + 12\hat{k}$

15. The value of $\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}\right)$ is:

(1) $\frac{4}{3}$ (2) 2

(3) $\frac{7}{3}$ (4) $\frac{5}{3}$

Ans. (4)

Sol. $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^3+6k^2+11k+5}{(k+3)!}$

$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^3+6k^2+11k+6-1}{(k+3)!}$

$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{(k+1)(k+2)(k+3)-1}{(k+3)!}$

$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{(k+1)(k+2)(k+3)}{(k+3)!} - \frac{1}{(k+3)!}$

$= \lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{k!} - \frac{1}{(k+3)!}\right)$

$= \lim_{n\to\infty}\left(\frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \ldots + \frac{1}{n!} - \frac{1}{4!} - \frac{1}{5!} - \frac{1}{6!} - \ldots - \frac{1}{(n+3)!}\right)$

$= \frac{1}{1} + \frac{1}{2} + \frac{1}{6} = \frac{10}{6} = \frac{5}{3}$

16. The integral $80\int_0^{\pi/4}\left(\frac{\sin\theta+\cos\theta}{9+16\sin 2\theta}\right)d\theta$ is equal to:

(1) $3\log_e 4$ (2) $6\log_e 4$

(3) $4\log_e 3$ (4) $2\log_e 3$

Ans. (3)

Sol. $I = 80\int_0^{\pi/4}\left(\frac{\sin\theta+\cos\theta}{9+16(2\sin\theta\cos\theta)}\right)d\theta$

$= 80\int_0^{\pi/4}\frac{\sin\theta+\cos\theta}{9-16(1-2\sin\theta\cos\theta-1)}d\theta$

$= 80\int_0^{\pi/4}\frac{\sin\theta+\cos\theta}{9+16-16(\sin\theta-\cos\theta)^2}d\theta$

Let $\sin\theta - \cos\theta = t$

$(\cos\theta + \sin\theta)d\theta = dt$

$= 80\int_{-1}^{0}\frac{dt}{25-16t^2}$

$= \frac{80}{16}\int_{-1}^{0}\frac{dt}{\left(\frac{5}{4}\right)^2 - t^2}$

$= \frac{5}{2\left(\frac{5}{4}\right)}\left[\ln\left|\frac{\frac{5}{4}+t}{\frac{5}{4}-t}\right|\right]_{-1}^{0}$

$= 2\ln(1) + 4\ln 3$

$= 4\ln 3$

17. Let $L_1: \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2}$ and $L_2: \frac{x+1}{-1} = \frac{y-2}{2} = \frac{z}{1}$ be two lines.

Let L_3 be a line passing through the point (α, β, γ) and be perpendicular to both L_1 and L_2. If L_3 intersects L_1, then $|5\alpha - 11\beta - 8\gamma|$ equals:

(1) 18 (2) 16

(3) 25 (4) 20

Ans. (3)

Sol. DR's of $L_3 = \vec{m}\times\vec{n} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ -1 & 2 & 1\end{vmatrix}$

$= -5\hat{i} - 3\hat{j} + \hat{k}$

$L_3: \frac{x-\alpha}{-5} = \frac{y-\beta}{-3} = \frac{z-\gamma}{1} = \lambda$

$A(-5\lambda+\alpha, -3\lambda+\beta, \lambda+\gamma)$

$L_1: \frac{x-1}{1} = \frac{y-2}{-1} = \frac{z-1}{2} = k$

$B(k+1, -k+2, 2k+1)$

Now

$-5\lambda + \alpha = k+1 \Rightarrow \alpha = 5\lambda + k + 1$

$-3\lambda + \beta = -k+2 \Rightarrow \beta = 3\lambda - k + 2$

$\lambda + \gamma = 2k+1 \Rightarrow \gamma = -\lambda + 2k + 1$

$|5\alpha - 11\beta - 8\gamma| = |-25|$

$= 25$

١٨. Let $x_١, x_٢, ..., x_{١٠}$ be ten observations such that $\sum_{i=١}^{١٠}(x_i - ٢) = ٣٠$, $\sum_{i=١}^{١٠}(x_i - \beta)^٢ = ٩٨$, $\beta > ٢$ and their variance is $\frac{٤}{٥}$. If μ and $\sigma^٢$ are respectively the mean and the variance of $٢(x_١ - ١) + ٤\beta$, $٢(x_٢ - ١) + ٤\beta$, ..., $٢(x_{١٠} - ١) + ٤\beta$, then $\frac{\beta\mu}{\sigma^2}$ is equal to :

(١) ١٠٠ (٢) ١١٠

(٣) ١٢٠ (٤) ٩٠

Ans. (١)

Sol. $\sum_{i=1}^{10} x_i = ٥٠$, $\Rightarrow$ mean $= ٥$

Variance $= \frac{٤}{٥} = \frac{\sum x_i^٢}{١٠} - \left(\frac{\sum x_i}{١٠}\right)^٢$

$\frac{٤}{٥} = \frac{\sum x_i^٢}{١٠} - ٢٥$

$\Rightarrow \sum x_i^٢ = ٢٥٨$ ....(١)

Now $\sum_{i=١}^{١٠}(x_i - \beta)^2 = ٩٨$

$\sum_{i=١}^{١٠} x_i^2 - 2\beta \sum x_i + 10\beta^2 = ٩٨$

$٢٥٨ - ٢\beta(٥٠) + ١٠\beta^2 = ٩٨$

$(\beta - ٨)(\beta - ٢) = ٠$

$\beta = 8$ or $\beta = ٢$ (as $\beta > ٢$)

$\Rightarrow \beta = 8$ ....(٢)

Now as per the question

$2(x_1 - 1) + 4\beta, 2(x_2 - 1) + 4\beta, ...2(x_{10} - 1) + 4\beta$

can be simplified to

$2x_1 + 30, 2x_2 + 30, ..., 2x_{10} + 30$ using eq. (٢)

$\mu = 2(5) + 30 = 40$ ....(٣)

$\sigma^2 = 4\left(\frac{٤}{٥}\right) = \frac{١٦}{٥}$

$\therefore \frac{\beta\mu}{\sigma^2} = \frac{٨ \times ٤٠}{١٦/٥} = ١٠٠$

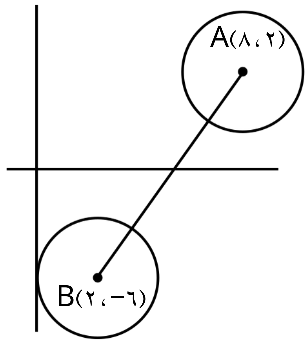
١٩. Let $|z_1 - ٨ - ٢i| \le ١$ and $|z_٢ - ٢ + ٦i| \le ٢$, $z_١, z_٢ \in C$. Then the minimum value of $|z_١ - z_٢|$ is :

(١) ٣ (٢) ٧

(٣) ١٣ (٤) ١٠

Ans. (٢)

Sol.



$\because AB = \sqrt{١٠٠} = ١٠$

$\Rightarrow |Z_١ - Z_٢|_{min} = ١٠ - ٢ - ١ = ٧$

٢٠. Let $A = [a_{ij}] = \begin{bmatrix} \log_٥ ١٢٨ & \log_٤ ٥ \\ \log_٥ ٨ & \log_٤ ٢٥ \end{bmatrix}$.

If A_{ij} is the cofactor of a_{ij}, $C_{ij} = \sum_{k=١}^{٢} a_{ik} A_{jk}$, $١ \le i$, $j \le ٢$, and $C = [C_{ij}]$, then $٨|C|$ is equal to :

(١) ٢٦٢ (٢) ٢٨٨

(٣) ٢٤٢ (٤) ٢٢٢

Ans. (٣)

$|A| = \frac{١١}{٢}$

$C_{١١} = \sum_{k=١} a_{١k} A_{١k} = a_{١١} A_{١١} + a_{١٢} A_{١٢} = |A| = \frac{١١}{٢}$

$C_{١٢} = \sum_{k=١} a_{١k} A_{٢k} = ٠$

$C_{٢١} = \sum_{k=١} a_{٢k} A_{١k} = ٠$

$C_{٢٢} = \sum_{k=١} a_{٢k} A_{٢k} = |A| = \frac{١١}{٢}$

$C = \begin{bmatrix} ١١/٢ & ٠ \\ ٠ & ١١/٢ \end{bmatrix}$

$|C| = \frac{١٢١}{٤}$

$٨|C| = ٢٤٢$

SECTION-B

21. Let $f:(0,\infty)\to\mathbb{R}$ be a twice differentiable function. If for some $a\neq 0$, $\int_0^1 f(\lambda x)\,d\lambda = af(x)$, $f(1)=1$ and $f(16)=\frac{1}{8}$, then $16-f'\left(\frac{1}{16}\right)$ is equal to ________.

Ans. (112)

Sol. $\int_0^1 f(\lambda x)\,d\lambda = af(x)$

$\lambda x = t$

$d\lambda = \frac{1}{x}dt$

$\frac{1}{x}\int_0^x f(t)\,dt = af(x)$

$\int_0^x f(t)\,dt = axf(x)$

$f(x) = a(xf'(x)+f(x))$

$(1-a)f(x) = a\cdot xf'(x)$

$\frac{f'(x)}{f(x)} = \left(\frac{1-a}{a}\right)\frac{1}{x}$

$\ln f(x) = \frac{1-a}{a}\ln x + c$

$x=1,\ f(1)=1 \Rightarrow c=0$

$x=16,\ f(16)=\frac{1}{8}$

$\frac{1}{8}=(16)^{\frac{1-a}{a}} \Rightarrow -3=\frac{4-4a}{a} \Rightarrow a=4$

$f(x)=x^{-\frac{3}{4}}$

$f'(x)=-\frac{3}{4}x^{-\frac{7}{4}}$

$\Rightarrow 16-f'\left(\frac{1}{16}\right)$

$=16+\frac{3}{4}\left(2^4\right)^{7/4}$

$=16+96=112$

22. Let $S=\{m\in\mathbb{Z}: A^{m^2}+A^m=3I-A^{-6}\}$, where $A=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$. Then $n(S)$ is equal to ________.

Ans. (2)

Sol. $A=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}$

$A^2=\begin{bmatrix}3 & -2\\ 2 & -1\end{bmatrix}, A^3=\begin{bmatrix}4 & -3\\ 3 & -2\end{bmatrix}, A^4=\begin{bmatrix}5 & -4\\ 4 & -3\end{bmatrix}$

and so on

$A^{-6}=\begin{bmatrix}-5 & 6\\ -6 & 7\end{bmatrix}$

$A^m=\begin{bmatrix}m+1 & -m\\ m & -m+1\end{bmatrix}$,

$A^{m^2}=\begin{bmatrix}m^2+1 & -m^2\\ m^2 & -m^2+1\end{bmatrix}$

$A^{m^2}+A^m=3I-A^{-6}$

$\begin{bmatrix}m^2+m+2 & -m^2-m\\ m^2+m & -m^2-m+2\end{bmatrix}$

$=3\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-5 & 6\\ -6 & 7\end{bmatrix}$

$=\begin{bmatrix}8 & -6\\ 6 & -4\end{bmatrix}$

$m^2+m+2=8$

$m^2+m-6=0 \Rightarrow m=-3, 2$

$n(S)=2$

23. Let $[t]$ be the greatest integer less than or equal to t. Then the least value of $p\in\mathbb{N}$ for which

$\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$

is equal to ________.

Ans. (24)

Sol. $\lim_{x\to 0^+}\left(x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+\ldots+\left[\frac{p}{x}\right]\right)-x^2\left(\left[\frac{1}{x^2}\right]+\left[\frac{2^2}{x^2}\right]+\ldots+\left[\frac{9^2}{x^2}\right]\right)\right)\geq 1$

$(1+2+\ldots\ldots+p)-(1^2+2^2+\ldots\ldots9^2)\geq 1$

$\frac{p□p□١□}{٢} □ \frac{١٠.١٠.١٩}{٦} □١$

p(p + ١) □٥٧٢

Least natural value of p is ٢٤

٢٤. The number of ٦-letter words, with or without meaning, that can be formed using the letters of the word MATHS such that any letter that appears in the word must appear at least twice, is ٤ ______.

Ans. (١٤٠٥)

Sol. (i) Single letter is used, then no. of words = ٥

(ii) Two distinct letters are used, then no. of words

$^{٥}C_2 □□ \frac{٦!}{٢!٤!} □ 2 □ \frac{٦!}{٣!٣!} □□ ١٠□٣٠ □٢٠□ ٥٠٠$

(iii) Three distinct letters are used, then no. of words

$^{٥}C_{٣} □ \frac{٦!}{٢!٢!٢!} □٩٠٠$

Total no. of words = ١٤٠٥

٢٥. Let S □ □x : cos□ sin□ x □ sin□ □٢x □١ □□.

Then $\sum_{x□S} □٢x □١□^٢$ is equal to ________.

Ans. (٥)

Sol. $cos^{-١}x = □ + sin^{-١}x + sin^{-١}(٢x + ١)$

$٢cos^{-١}x - sin^{-١}(٢x + ١) = \frac{3□}{٢}$

$٢□ - □ = \frac{3□}{٢}$ where $cos^{-١}x = □, sin^{-١}(٢x + ١) = □$

$٢□ = \frac{٣□}{٢} + □$

cos٢□ = sin □

٢cos□ − ١ = sin □

$٢x^٢ - ١ = ٢x + ١$

$x^٢ - x - ١ = ٠$

$□x = \frac{١ □\sqrt{٥}}{٢}, □□x□ \frac{١ □\sqrt{٥}}{٢}$ rejected

$□ ٤x^٢ - ٤x = ٤$

$(٢x - ١)^٢ = ٥$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 29th JANUARY 2025) TIME : 9 : 00 AM TO 12 : 00 NOON

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

٢٦. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : Choke coil is simply a coil having a large inductance but a small resistance. Choke coils are used with fluorescent mercury-tube fittings. If household electric power is directly connected to a mercury tube, the tube will be damaged.
Reason (R) : By using the choke coil, the voltage across the tube is reduced by a factor $\left(R/\sqrt{R^{٢}+\omega^{٢}L^{٢}}\right)$, where ω is frequency of the supply across resistor R and inductor L. If the choke coil were not used, the voltage across the resistor would be the same as the applied voltage.
In the light of the above statements, choose the most appropriate answer from the options given below:
(١) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(٢) (A) is false but (R) is true.
(٣) Both (A) and (R) are true and (R) is the correct explanation of (A).
(٤) (A) is true but (R) is false.

Ans. (٣)
Sol. A: Correct
B : Correct with correct explanation

٢٧. Two projectiles are fired with same initial speed from same point on ground at angles of (٤٥° – □) and (٤٥° + □), respectively, with the horizontal direction. The ratio of their maximum heights attained is :

(١) $\frac{١ □ tan□}{١ □ tan□}$ (٢) $\frac{١ □ sin□}{١ □ sin□}$
(٣) $\frac{١ □ sin □}{١ □_{٢} □}$ (٤) $\frac{١ □ sin □}{١ □_{٢} □}$

Ans. (٣) sin ٢ sin ٢

Sol. $H_{Max} □ \frac{(u\sin□)^{٢}}{٢g}$

$\frac{(H_{max})_{١}}{(H_{max})_{٢}} □ \frac{u^{٢}\sin^{٢}(٤٥ □ □)}{u^{٢}\sin^{٢}(٤٥ □ □)}$

$□ \frac{□\frac{١}{\sqrt{٢}}\cos□ □ \frac{١}{\sqrt{٢}}\sin□□^{٢}}{□\frac{١}{\sqrt{٢}}\cos□ □ \frac{١}{\sqrt{٢}}\sin□□}$

$□ \frac{1 □ \sin □}{1 □ ٢ □}$

٢٨. An electric dipole of mass m, charge q, and length l is placed in a uniform electric field $E□E\hat{i}$.
When the dipole is rotated slightly from its equilibrium position and released, the time period of its oscillations will be :

(١) $\frac{١}{٢□}\sqrt{\frac{٢ml}{qE}}$ (٢) $٢□\sqrt{\frac{ml}{qE}}$
(٣) $\frac{١}{٢□}\sqrt{\frac{ml}{٢qE}}$ (٤) $٢□\sqrt{\frac{ml}{٢qE}}$

Ans. (٤)
Sol. $I□^{٢}□ = q□E□$

$٢m□\frac{\ell}{٢}□^{٢} □2□q\ell E$

$□2□\frac{٢qE}{m\ell}$

$T □2□\sqrt{\frac{m\ell}{٢qE}}$

٢٩. The pair of physical quantities not having same dimensions is :
(١) Torque and energy
(٢) Surface tension and impulse
(٣) Angular momentum and Planck's constant
(٤) Pressure and Young's modulus

Ans. (٢)

Sol. [L] = [E][t] [I]

[L] = [h] [P] = [Y] Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

٣٠. Assertion (A) : Time period of a simple pendulum is longer at the top of a mountain than that at the base of the mountain. Reason (R) : Time period of a simple pendulum decreases with increasing value of acceleration due to gravity and vice-versa. In the light of the above statements, choose the most appropriate answer from the options given below: (١) Both (A) and (R) are true but (R) is not the correct explanation of (A). (٢) Both (A) and (R) are true and (R) is the correct explanation of (A). (٣) (A) is true but (R) is false. (٤) (A) is false but (R) is true.

Ans. (٢)

Sol. As h increases, g decreases, T increases

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g = \frac{gR^{٢}}{(R+h)^{٢}}$$

٣١. The expression given below shows the variation of velocity (v) with time (t), $v = \frac{At^{٢} + Bt}{C + t}$. The dimension of ABC is :

(١) [M⁰L٢T⁻٣]

(٢) [M⁰L١T⁻٣]

(٣) [M⁰L١T⁻٢]

(٤) [M⁰L٢T⁻٢]

Ans. (١)

Sol. [LT⁻¹] = [A][T٢] = $\frac{[B][T]}{[C]+[T]}$

[C] = [T]

[A] = [LT⁻³]

[B] = [LT⁻¹]

[ABC] = [L٢T⁻³]

٣٢. Consider $I_١$ and $I_٢$ are the currents flowing simultaneously in two nearby coils ١ & ٢, respectively. If $L_١$ = self inductance of coil ١, $M_{١٢}$ = mutual inductance of coil ١ with respect to coil ٢, then the value of induced emf in coil ١ will be

(١) $\varepsilon_١ = -L_١\frac{dI_١}{dt} - M_{١٢}\frac{dI_٢}{dt}$

(٢) $\varepsilon_١ = -L_١\frac{dI_١}{dt} + M_{١٢}\frac{dI_١}{dt}$

(٣) $\varepsilon_١ = -L_١\frac{dI_١}{dt} - M_{١٢}\frac{dI_٢}{dt}$

(٤) $\varepsilon_١ = -L_١\frac{dI_٢}{dt} - M_{١٢}\frac{dI_١}{dt}$

Ans. (٣)

Sol. $\phi_١ = L_١I_١ + M_{١٢}I_٢$

$$\varepsilon_١ = -\frac{d\phi_١}{dt} = -L_١\frac{dI_١}{dt} - M_{١٢}\frac{dI_٢}{dt}$$

٣٣. At the interface between two materials having refractive indices $n_١$ and $n_٢$, the critical angle for reflection of an em wave is $\theta_{١C}$. The $n_٢$ material is replaced by another material having refractive index $n_٣$, such that the critical angle at the interface between $n_١$ and $n_٣$ materials is $\theta_{٢C}$. If $n_٣ > n_٢ > n_١$; $\frac{n_٢}{n_٣} = \frac{٢}{٥}$ and $\sin\theta_{٢C} - \sin\theta_{١C} = \frac{١}{٢}$, then $\theta_{١C}$ is

(١) $\sin^{-1}\left(\frac{١}{٦n_١}\right)$

(٢) $\sin^{-1}\left(\frac{٢}{٣n_١}\right)$

(٣) $\sin^{-1}\left(\frac{٥}{٦n_١}\right)$

(٤) $\sin^{-1}\left(\frac{١}{٣n_١}\right)$

NTA Ans. (٤)

Sol. $\sin\theta_{١C} = \frac{n_١}{n_٢}$

$\sin\theta_{٢C} = \frac{n_١}{n_٣}$

$\sin\theta_{٢C} - \sin\theta_{١C} = \frac{١}{٢}$

$n_1 \frac{n_2}{n} \quad \frac{n_1}{n_1} \quad \frac{1}{2}$

$n_1 \frac{n_2}{3} \quad \frac{n_2}{n_1 \; 2}$

$n_1 \frac{2}{5} \quad 1 \quad \frac{n_2}{2}$

$\frac{n_1}{n} \quad \frac{5}{12}$

$\frac{1}{5}$

$\sin \frac{}{6}$

٣٤. Consider a straight wire of a circular cross-section (radius a) carrying a steady current I. The current is uniformly distributed across this cross-section. The distances from the centre of the wire's cross-section at which the magnetic field (inside the wire, outside the wire) is half of the maximum possible magnetic field, any where due to the wire, will be

(١) ($a/4$, $3a/2$) (٢) ($a/2$, $2a$)

(٣) ($a/2$, $3a$) (٤) ($a/4$, $2a$)

Ans. (٢)

Sol. Maximum possible magnetic field is at the surface

$$B_{max} = \frac{\mu_0 I}{2\pi a}$$

$$\frac{B_{max}}{2} = \frac{\mu_0 I}{4\pi a}$$

It can be obtained inside as well as outside the wire

For inside,

$$\frac{\mu_0 I}{4\pi a} = \frac{\mu_0 I r}{2\pi a^2}$$

$$\Rightarrow r = \frac{a}{2}$$

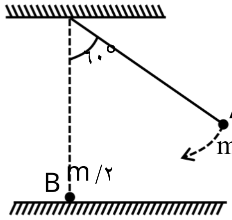
For outside

$$\frac{\mu_0 I}{4\pi a} = \frac{\mu_0 I}{2\pi r}$$

$\Rightarrow r = 2a$

Correct answer $\left(\frac{a}{2}, 2a\right)$

٣٥. As shown below, bob A of a pendulum having massless string of length 'R' is released from ٦٠° to the vertical. It hits another bob B of half the mass that is at rest on a friction less table in the centre. Assuming elastic collision, the magnitude of the velocity of bob A after the collision will be (take g as acceleration due to gravity)

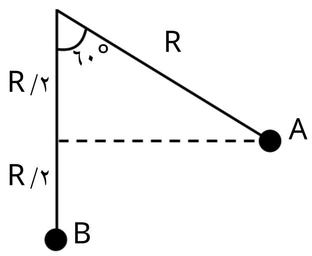


(١) $\frac{1}{3}\sqrt{Rg}$ (٢) $\sqrt{Rg}$

(٣) $\frac{4}{3}\sqrt{Rg}$ (٤) $\frac{2}{3}\sqrt{Rg}$

Ans. (١)

Sol.



Velocity of a just before hitting:

$$u = \sqrt{2g\frac{R}{2}} = \sqrt{gR}$$

Just after collision, let velocity of A and B are v_1 and v_2 respectively

$\therefore$ by COM:

$$mu = mv_1 + \frac{m}{2} v_2$$

$$2v_1 + v_2 = 2u \qquad \text{....(i)}$$

$$e = 1 = \frac{v_2 - v_1}{u}$$

$$\Rightarrow v_2 - v_1 = u \qquad \text{....(ii)}$$

From (i) – (ii)

$$\Rightarrow 3v_1 = u \Rightarrow v_1 = \frac{u}{3} = \frac{1}{3}\sqrt{gR}$$

٣٦. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Emission of electrons in photoelectric effect can be suppressed by applying a sufficiently negative electron potential to the photoemissive substance.

Reason (R) : A negative electric potential, which stops the emission of electrons from the surface of a photoemissive substance, varies linearly with frequency of incident radiation.

In the light of the above statements, choose the most appropriate answer from the options given below:

(١) (A) is false but (R) is true.

(٢) (A) is true but (R) is false.

(٣) Both (A) and (R) are true and (R) is the correct explanation of (A).

(٤) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Ans. (٤)

Sol. (A) : True

(B) : True but not correct explanation

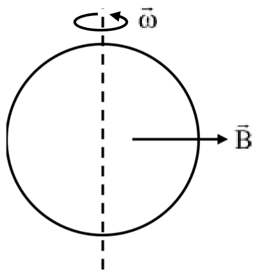
٣٧. A coil of area A and N turns is rotating with angular velocity ω in a uniform magnetic field $\vec{B}$ about an axis perpendicular to $\vec{B}$. Magnetic flux φ and induced emf ε across it, at an instant when $\vec{B}$ is parallel to the plane of coil, are :

(١) φ = AB, ε = ٠ (٢) φ = ٠, ε = NABω

(٣) φ = ٠, ε = ٠ (٤) φ = AB, ε = NABω

Ans. (٢)

Sol.



φ = BAN.cos(ωt)

$\varepsilon = \frac{-d\phi}{dt}$ = BAωN.sin(ωt)

When B is parallel to plane ωt = $\frac{\pi}{٢}$

∴ φ = ٠, ε = BAωN

٣٨. The fractional compression $\left(\frac{\Delta V}{V}\right)$ of water at the depth of ٢.٥ km below the sea level is __________ %. Given, the Bulk modulus of water = ٢ × ١٠⁹ Nm⁻², density of water = ١٠³ kg m⁻³, acceleration due to gravity = g = ١٠ ms⁻².

(١) ١.٧٥ (٢) ١.٠

(٣) ١.٥ (٤) ١.٢٥

Ans. (٤)

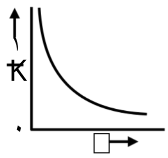
Sol. $B = \frac{\rho gh}{\left(\frac{\Delta v}{v}\right)}$

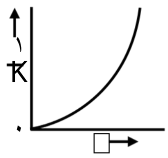
$\frac{\Delta v}{v} \times ١٠٠ = \frac{\rho gh}{B} \times ١٠٠$

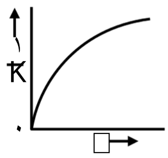
$\frac{١٠٠٠ \times ١٠ \times ٢.٥ \times ١٠^٣}{٢ \times ١٠^٩} \times ١٠٠\%$

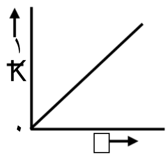
= ١.٢٥ %

٣٩. If λ and K are de Broglie Wavelength and kinetic energy, respectively, of a particle with constant mass. The correct graphical representation for the particle will be :-

(١) 

(٢) 

(٣) 

(٤) 

Ans. (٢)

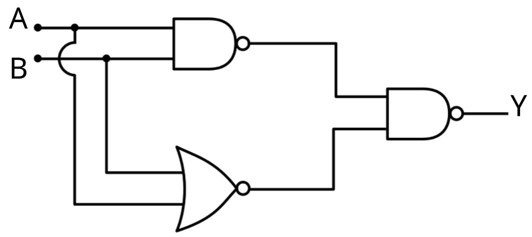
Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{٢mK}}$

$\Rightarrow \frac{1}{\lambda^2} = \left(\frac{٢m}{h^٢}\right) k$

Y = cx ٢

Upward facing parabola passing through origin.

٤٠.



For the circuit shown above، equivalent GATE is :

(١) OR gate (٢) NOT gate

(٣) AND gate (٤) NAND gate

Ans. (١)

Sol.

A	B	Y
٠	٠	٠
٠	١	١
١	٠	١
١	١	١

□ OR Gate

٤١. A body of mass 'm' connected to a massless and unstretchable string goes in verticle circle of radius 'R' under gravity g. The other end of the string is fixed at the center of circle. If velocity at top of circular path is $\sqrt{ngR}$، where، n□١، then ratio of kinetic energy of the body at bottom to that at top of the circle is

(١) $\frac{n}{n□٤}$ (٢) $\frac{n□٤}{n}$

(٣) $\frac{n}{n٢□٤}$ (٤) $\frac{n٢□٤}{n٢}$

Ans. (٤)

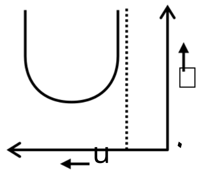
Sol. $V_{Top} □ \sqrt{n٢gR}$

$V_{Bottom} □ \sqrt{n٢gR□٤gR}$

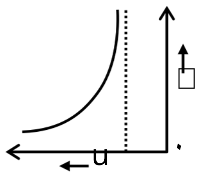
Ratio $□ \frac{n٢□٤}{n٢}$

٤٢. Let u and v be the distances of the object and the image from a lens of focal length f. The correct graphical representation of u and v for a convex lens when |u| ‹ f، is

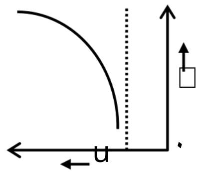
(١)



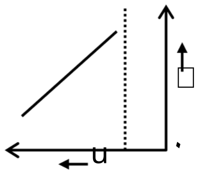
(٢)



(٣)

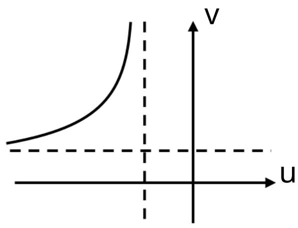


(٤)



Ans. (٢)

Sol. (u + f)(v − f) = f ٢



٤٣. Match List-I with List-II.

	List-I		List-II
(A)	Electric field inside (distance r ‹ ٠ from center) of a uniformly charged spherical shell with surface charge density □، and radius R.	(I)	□ / □.
(B)	Electric field at distance r ‹ ٠ from a uniformly charged infinite plane sheet with surface charge density □□	(II)	□ / ٢□ □
(C)	Electric field outside (distance r ‹ ٠ from center) of a uniformly charged spherical shell with surface charge density □، and radius R	(III)	□□
(D)	Electric field between ٢ oppositely charged infinite plane parallel sheets with uniform surface charge density □.	(IV)	$\frac{□}{□}\frac{R}{r٢}$ □

Choose the correct answer from the options given below :

(١) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

(٢) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(٣) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(٤) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Ans. (٤)

Sol. (A) □ ٠ (III)

(B) □ $\frac{□}{٢□}$ (II)

(C) □ $\frac{□R^{٢}}{□r^{٢}}$ (No row matching)

(D) □ $\frac{□}{□}$ (I)

٤٤. The workdone in an adiabatic change in an ideal gas depends upon only :

(١) change in its pressure

(٢) change in its specific heat

(٣) change in its volume

(٤) change in its temperature

Ans. (٤)

Sol. $□W = -□U = -nC_V□T$

٤٥. Given below are two statements : one is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Electromagnetic waves carry energy but not momentum.

Reason (R) : Mass of a photon is zero.

In the light of the above statements, choose the most appropriate answer from the options given below :

(١) (A) is true but (R) is false.

(٢) (A) is false but (R) is true.

(٣) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(٤) Both (A) and (R) are true and (R) is the correct explanation of (A).

Ans. (٢)

Sol. Assertion is false because em waves have momentum.

SECTION-B

٤٦. The coordinates of a particle with respect to origin in a given reference frame is (١, ١, ١) meters. If a force of $F□\hat{i}□\hat{j}□\hat{k}$ acts on the particle, then the magnitude of torque (with respect to origin) in z-direction is ___________.

Ans. (٢)

Sol. $□ □ r □ F □ \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ ١ & ١ & ١ \\ ١ & □1 & ١ \end{vmatrix}$

$□ □ \hat{k}(□١□١) □ □٢\hat{k}$

$|□| □ ٢Nm$

٤٧. A container of fixed volume contains a gas at ٢٧°C. To double the pressure of the gas, the temperature of gas should be raised to __________ °C.

Ans. (٣٢٧)

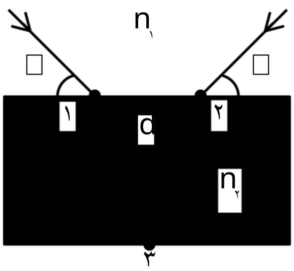
Sol. $\frac{P_١}{T_١} □ \frac{P_٢}{T_٢}$

$\frac{P}{٣٠٠} □ \frac{٢P}{T_٢}$

$T_٢ = ٦٠٠ K$

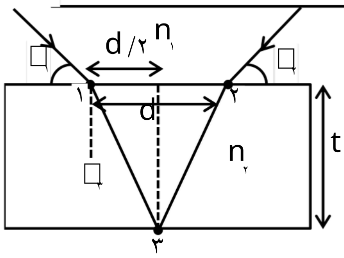
$T_٢ = ٣٢٧°C$

٤٨. Two lightbeams fall on a transparent material block at point ١ and ٢ with angle □ and □, respectively, as shown in figure. After refraction, the beams intersect at point ٣ which is exactly on the interface at other end of the block. Given : the distance between ١ and ٢, d□٤٣ cm and $□□□_٢ □ \cos^{-1}\left(\frac{n_٢}{٢n_١}\right)$, where refractive index of the block $n_٢$ refractive index of the outside medium $n_١$, then the thickness of the block is ___________ cm.



Ans. (٦)

Sol.



$n_١ \sin(٩٠ - □) = n_١ \sin □_٢$

$n_١ \cos □_١ = n_٢ \sin □_٣$

$n_١ \frac{n_٢}{n_١} □ n_٢ \sin □_٣$

$\frac{١}{٢} □ \sin □_٣ ، □_٣ = ٣٠$

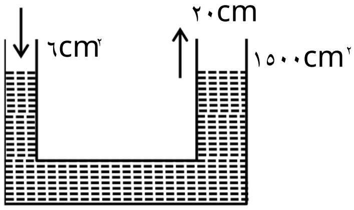
$\tan ٣٠ □ \frac{d}{٢(t)}$

$t □ \frac{d\sqrt{٣}}{٢} □ \frac{٤\sqrt{3} □ \sqrt{٣}}{٢} \text{cm} □ ٦\text{cm}$

٤٩. In a hydraulic lift، the surface area of the input piston is ٦ cm² and that of the output piston is ١٥٠٠ cm². If ١٠٠ N force is applied to the input piston to raise the output piston by ٢٠ cm، then the work done is ____________ kJ.

Ans. (٥)

Sol.



$\frac{F_١}{A_١} □ \frac{F_٢}{A_٢} ، \frac{١٠٠}{٦} □ \frac{F}{١٥٠٠} ، F □ \frac{٥٠}{٣} □ ١٥٠٠$

$F = ٥٠ \times ٥٠٠ = ٢٥ \times ١٠^٣ N$

$□ □ F.S □ ٢٥ □ ١٠^٣ □ \frac{٢٠}{١٠٠}$

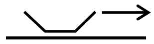
$= ٥ \times ١٠^٣ = ٥ \text{ kJ}$

٥٠. The maximum speed of a boat in still water is ٢٧ km/h. Now this boat is moving downstream in a river flowing at ٩ km/h. A man in the boat throws a ball vertically upwards with speed of ١٠ m/s. Range of the ball as observed by an observer at rest on the river bank is ____________ cm. (Take g = ١٠ m/s²)

Ans. (٢٠٠٠)

Sol.

$v_b = ٩ + ٢٧ = ٣٦ \text{ km/hr}$



$v_b □ ٣٦ □ \frac{١٠٠٠}{٣٦٠٠٠} = ١٠ \text{ m/sec}$

$\text{Time of flight} = \frac{٢ □ ١٠}{١٠} = ٢ \text{ sec}$

$\text{Range} = ١٠ \times ٢ = ٢٠ \text{m} = ٢٠٠٠ \text{ cm}$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 29th JANUARY 2025) TIME : 9 : 00 AM TO 12 : 00 NOON

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION–A

٥١. Total number of nucleophiles from the following is :–

$NH_٣$, PhSH, $(H_٣C)_٢S$, $H_٢C=CH_٢$, $OH^{\ominus}$, $H_٣O^{\oplus}$, $(CH_٣)_٢CO$, NCH٣

(١) ٥ (٢) ٤

(٣) ٧ (٤) ٦

Ans. (١)

Sol. Total five nucleophiles are present $NH_٣$, PhSH, $(H_٣C)_٢S$, $CH_٢=CH_٢$, $OH^{\ominus}$

٥٢. The standard reduction potential values of some of the p-block ions are given below. Predict the one with the strongest oxidising capacity.

(١) $E^{\ominus}_{Sn^{٤+}/Sn^{٢+}} = ١.١٥V$ (٢) $E^{\ominus}_{Tl^{3+}/Tl} = ١.٢٦V$

(٣) $E^{\ominus}_{Al^{٣+}/Al} = -١.٦٦V$ (٤) $E^{\ominus}_{Pb^{٤+}/Pb^{٢+}} = ١.٦٧V$

Ans. (٤)

Sol. Standard reduction potential value (+ve) increases oxidising capacity increases.

٥٣. The molar conductivity of a weak electrolyte when plotted against the square root of its concentration, which of the following is expected to be observed?

(١) A small decrease in molar conductivity is observed at infinite dilution.

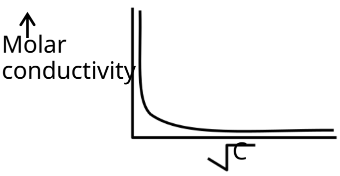
(٢) A small increase in molar conductivity is observed at infinite dilution.

(٣) Molar conductivity increases sharply with increase in concentration.

(٤) Molar conductivity decreases sharply with increase in concentration.

Ans. (٤)

Sol.



٥٤. At temperature T, compound $AB_٢(g)$ dissociates as $AB_٢(g) \rightleftharpoons AB(g) + \frac{١}{٢}B_٢(g)$ having degree of dissociation x (small compared to unity). The correct expression for x in terms of Kp and p is

(١) $٣\sqrt{\frac{٢K_p}{p}}$ (٢) $٤\sqrt{\frac{٢K_p}{p}}$

(٣) $٣\sqrt{\frac{٢K_p^٢}{p}}$ (٤) $\sqrt{K_p}$

Ans. (٣)

Sol. $AB_{٢(g)} \rightleftharpoons AB(g) + \frac{١}{٢}B_{٢(g)}$

$$t_{eq}\ \frac{(١-x)}{1+\frac{x}{٢}}P \quad \frac{xP}{1+\frac{x}{٢}} \quad \frac{\left(\frac{x}{2}\right)P}{1+\frac{x}{٢}}$$

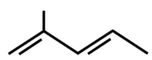
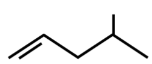
$\because$ x is very small so $1+\frac{x}{٢} \simeq 1$ and $١-x \simeq ١$

$$\therefore k_p = \frac{(xp)\left(\frac{xp}{2}\right)^{\frac{١}{٢}}}{P}$$

$$\therefore k_p^٢ = \frac{x^٣ P}{٢}$$

$$x = \sqrt[٣]{\frac{٢k_P^٢}{P}}$$

٥٥. Match List-I with List-II.

	List-I (Structure)		List-II (IUPAC Name)
(A)	$H_٣C-CH_٢-CH-CH_٢-CH-C_٢H_٥$ (branches: $C_٢H_٥$, $CH_٣$)	(I)	٤-Methylpent-١-ene
(B)	$(CH_٣)_٢C(C_٣H_٧)_٢$	(II)	٣-Ethyl-٥-methylheptane
(C)		(III)	٤,٤-Dimethylheptane
(D)		(IV)	٢-Methyl-١,٣-pentadiene

Choose the correct answer from the options given below:

(١) (A)-(III), (B)-(II), (C)-(IV), (D)-(I) (٢) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(٣) (A)-(II), (B)-(III), (C)-(IV), (D)-(I) (٤) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

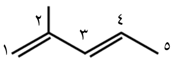
Ans. (٣)

Sol. (A) CH٣-CH٢-CH-CH٢-CH-CH٢-CH٣ (carbons numbered ١ to ٧), with CH٢-CH٣ and CH٣ branches

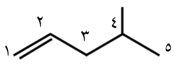
٣-Ethyl-٥-methylheptane

(B) (CH٣)٢C(C٣H٧)٢

CH٣-C(CH٣)-CH٢-CH٢-CH٣ with CH٢-CH٢-CH٣ ٤,٤-Dimethylheptane

(C) 

٢-Methyl-١,٣-pentadiene

(D) 

٤-Methylpent-١-ene

٥٦. Choose the correct statements.

(A) Weight of a substance is the amount of matter present in it.

(B) Mass is the force exerted by gravity on an object.

(C) Volume is the amount of space occupied by a substance.

(D) Temperatures below ٠°C are possible in Celsius scale, but in Kelvin scale negative temperature is not possible.

(E) Precision refers to the closeness of various measurements for the same quantity.

(١) (B), (C) and (D) Only

(٢) (A), (B) and (C) Only

(٣) (A), (D) and (E) Only

(٤) (C), (D) and (E) Only

Ans. (٤)

Sol. Theory based

٥٧. The correct increasing order of stability of the complexes based on Δo value is:

(I) [Mn(CN)٦]٣- (II) [Co(CN)٦]٣-

(III) [Fe(CN)٦]٤- (IV) [Fe(CN)٦]٣-

(١) II > III > I > IV (٢) I > II > IV > III

(٣) IV > III > II > I (٤) III > II > IV > I

Ans. (٣)

Sol. (I) [Mn(CN)٦]٣- -١.٦ Δo

(II) [Co(CN)٦]٣- -١.٨ Δo

(III) [Fe(CN)٦]٤- -٢.٤ Δo

(IV) [Fe(CN)٦]٣- -٢.٠ Δo

I < II < IV < III

٥٨. Match List-I with List-II.

List-I (Complex)		List-II (Hybridisation & Magnetic characters)	
(A)	[MnBr٤]٢-	(I)	d٢sp٣ & diamagnetic
(B)	[FeF٦]٣-	(II)	sp٣d٢ & paramagnetic
(C)	[Co(C٢O٤)٣]٣-	(III)	sp٣ & diamagnetic
(D)	[Ni(CO)٤]	(IV)	sp٣ & paramagnetic

Choose the correct answer from the options given below:

(١) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(٢) (A)-(III), (B)-(I), (C)-(II), (D)-(IV) (٣) (A)-(IV), (B)-(I), (C)-(II), (D)-(III) (٤) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Ans. (٤)

Sol. (A) [MnBr٤]٢-

Mn+٢ → [Ar] ٣d٥

In presence of ligand field

→ [Ar] ↿ ↿ ↿ ↿ ↿ (٣d) □ (٤s) □□□ (٤p)

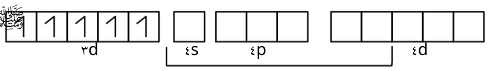
∴ sp٣ hybridization, paramagnetic in nature

(B) [FeF٦]

Fe ⇒ [Ar] ٣d

In presence of ligand field

⇒ [Ar]



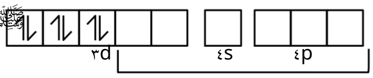
∴ sp٣d٢ hybridization، paramagnetic in nature

(C) [Co(C٢O٤)٣]

Co ⇒ [Ar] ٣d٦

In presence of ligand field

⇒ [Ar]



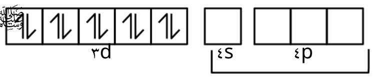
∴ d٢sp٣ hybridization، diamagnetic in nature

(D) [Ni(CO)٤]

Ni ⇒ [Ar] ٣d٨٤s٢

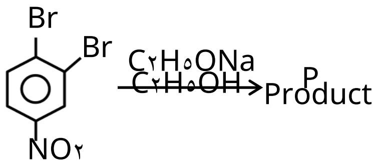
In presence of ligand field

⇒ [Ar]

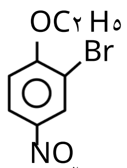


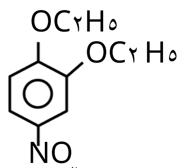
∴ sp٣ hybridization، diamagnetic in nature

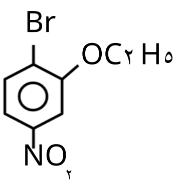
٥٩. In the following substitution reaction:

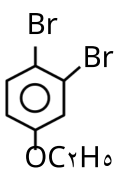


Product 'P' formed is:

(١)


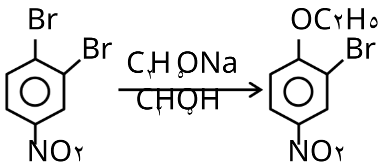
(٢)


(٣)


(٤)


Ans. (١)

Sol. It is an example of nucleophillic Aromatic substitution reaction.



٦٠. For a Mg | Mg٢⁺(aq) || Ag⁺(aq) | Ag the correct Nernst Equation is:

(1) $E_{cell} = E^{o}_{cell} - \frac{RT}{2F} \ln \frac{[Ag^{+}]}{[Mg^{2+}]}$

(٢) $E_{cell} = E^{o}_{cell} + \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

(٣) $E_{cell} = E^{o}_{cell} + \frac{RT}{2F} \ln \frac{[Mg^{2+}]}{[Ag^{+}]}$

(٤) $E_{cell} = E^{o}_{cell} - \frac{RT}{2F} \ln \frac{[Ag^{+}]^2}{[Mg^{2+}]}$

Ans. (٢)

Sol. According to Nernst equation :-

$$E = E^{\circ} - \frac{RT}{nF} \ln Q.$$

Cell reaction :-

$$Mg(s) + 2Ag^{+}_{(aq)} \rightleftharpoons 2Ag(s) + Mg^{2+}_{(aq)}$$

$$\therefore Q = \frac{[Mg^{2+}]}{[Ag^{+}]^2}$$

$$\therefore E_{cell} = E^{o}_{cell} - \frac{RT}{2F} \ln \left(\frac{[Mg^{2+}]}{[Ag^{+}]^2} \right)$$

٦١. The correct option with order of melting points of the pairs (Mn، Fe)، (Tc، Ru) and (Re، Os) is: (١) Fe > Mn، Ru > Tc and Re > Os (٢) Mn > Fe، Tc > Ru and Re > Os (٣) Mn > Fe، Tc > Ru and Os > Re (٤) Fe > Mn، Ru > Tc and Os > Re

Ans. (٣)

Sol. M.P. ⇒ Mn > Fe، Tc > Ru، Os > Re

NCERT based

٦٢. ١.٢٤ g of AX٢ (molar mass ١٢٤ g mol⁻¹) is dissolved in ١ kg of water to form a solution with boiling point of ١٠٠.٠١٥٦°C، while ٢٥.٤ g of AY٢ (molar mass ٢٥٠ g mol⁻¹) in ٢ kg of water constitutes a solution with a boiling point of ١٠٠.٠٢٦٠°C.

Kb(H٢O) = ٠.٥٢ K kg mol⁻¹

Which of the following is correct ؟

(١) $AX_٢$ and $AY_٢$ (both) are completely unionised.

(٢) $AX_٢$ and $AY_٢$ (both) are fully ionised.

(٣) $AX_٢$ is completely unionised while $AY_٢$ is fully ionised.

(٤) $AX_٢$ is fully ionised while $AY_٢$ is completely unionised.

Ans. (٤)

Sol. For $AX_٢$:– $\Delta Tb = Kb \times m \times i$

$٠.٠١٥٦ = ٠.٥٢ \times \frac{٠.٠١}{١} \times i_{AX_٢}$

$\Rightarrow i_{AX_٢} = ٣ \Rightarrow$ complete ionisation

For $AY_٢$:– $\Delta Tb = Kb \times m \times i$

$٠.٠٢٦ = ٠.٥٢ \times ٠.٠٥٠٨ \times i_{AY_٢}$

$\Rightarrow i_{AY_٢} \simeq ١ \Rightarrow$ complete unionisation

٦٣. ٥٠٠ J of energy is transferred as heat to ٠.٥ mol of Argon gas at ٢٩٨ K and ١.٠٠ atm. The final temperature and the change in internal energy respectively are :

Given : $R = ٨.٣\ J\ K^{-١}\ mol^{-١}$

(١) ٣٤٨ K and ٣٠٠ J

(٢) ٣٧٨ K and ٣٠٠ J

(٣) ٣٦٨ K and ٥٠٠ J

(٤) ٣٧٨ K and ٥٠٠ J

NTA Ans. (٤)

Sol. $q_p = n \times c_p \times \Delta T$

$\Rightarrow ٥٠٠ = ٠.٥ \times \frac{٥}{٢} \times ٨.٣ (T_f - ٢٩٨)$

$\Rightarrow T_f \simeq ٣٤٦.٢K$

$\Rightarrow \frac{\Delta H}{\Delta U} = \frac{C_p}{C_v} = \left(\frac{٥}{٣}\right)$

$\Rightarrow \Delta U = \frac{٣}{٥} \times ٥٠٠ = ٣٠٠ J$

٦٤. The reaction $A_٢ + B_٢ \rightarrow ٢AB$ follows the mechanism

$A_2 \underset{k_{-1}}{\overset{k_1}{\rightleftharpoons}} A + A \text{(fast)}$

$A + B_٢ \xrightarrow{k_٢} AB + B$ (slow)

$A + B \rightarrow AB$ (fast)

The overall order of the reaction is :

(١) ١.٥

(٢) ٣

(٣) ٢.٥

(٤) ٢

Ans. (١)

Sol. rate $= k_٢[A][B_٢]$ ……(١)

$\left(\frac{k_١}{k_{-١}}\right) = \frac{[A]^٢}{[A_2]}$

$\Rightarrow [A] = \sqrt{\frac{k_١}{k_{-١}}} \cdot \sqrt{[A_٢]}$

Substituting in (١), we get

Rate $= k_٢ \sqrt{\frac{k_١}{k_{-١}}} \cdot [A_2]^{\frac{١}{٢}}[B_٢]$

$\Rightarrow$ order $= \left(\frac{٣}{٢}\right) = ١.٥$

٦٥. If $a_٠$ is denoted as the Bohr radius of hydrogen atom, then what is the de-Broglie wavelength (λ) of the electron present in the second orbit of hydrogen atom؟ [n : any integer]

(١) $\frac{2a_0}{n\pi}$

(٢) $\frac{8\pi a_0}{n}$

(٣) $\frac{4\pi a_0}{n}$

(٤) $\frac{4n}{\pi a_0}$

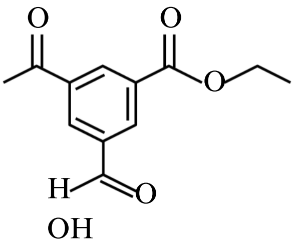
Ans. (٢)

Sol. $٢\pi r_n = n\lambda$

$٢\pi.(٤a_٠) = n\lambda$

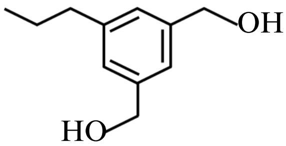
$= \lambda = \frac{٨\pi a_٠}{n}$

٦٦. The product (P) formed in the following reaction is :

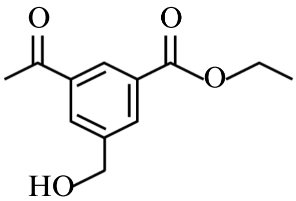


□Z□n□H□g□ Product (P)
HCl

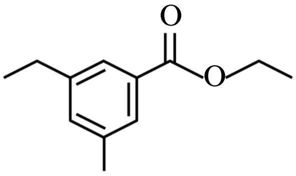
(١)



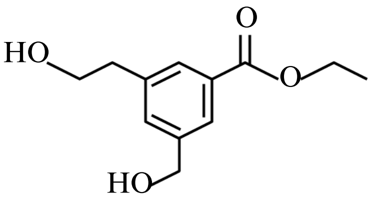
(٢)



(٣)

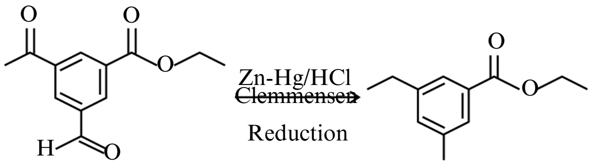


(٤)



Ans. (٣)

Sol.



٦٧. An element 'E' has the ionisation enthalpy value of ٣٧٤ kJ mol⁻¹. 'E' reacts with elements A، B، C and D with electron gain enthalpy values of –٣٢٨، –٣٤٩، –٣٢٥ and –٢٩٥ kJ mol⁻¹، respectively. The correct order of the products EA، EB، EC and ED in terms of ionic character is :

(١) EB < EA < EC < ED

(٢) ED < EC < EA < EB

(٣) EA < EB < EC < ED

(٤) ED < EC < EB < EA

Ans. (١)

Sol. Difference between I.E. & E.G.E increases، ionic character increases.

٦٨. Match List – I with List – II.

	List – I (Carbohydrate)	List – II (Linkage Source)
(A)	Amylose	(I) □C١–C٤، plant
(B)	Cellulose	(II) □C١–C٤، animal
(C)	Glycogen	(III) □C١–C٤، □–C١–C٦، plant□
(D)	Amylopectin	(IV) □–C١–C٤، plant□

Choose the correct answer form the options given below :

(١) (A)–(III)، (B)–(II)، (C)–(I)، (D)–(IV)

(٢) (A)–(IV)، (B)–(I)، (C)–(II)، (D)–(III)

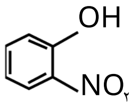
(٣) (A)–(II)، (B)–(III)، (C)–(I)، (D)–(IV)

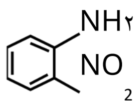
(٤) (A)–(IV)، (B)–(I)، (C)–(III)، (D)–(II)

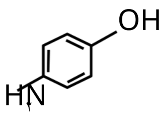
Ans. (٢)

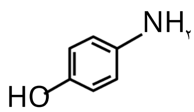
Sol. Informative

٦٩. The steam volatile compounds among the following are :

(A) 

(B) 

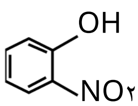
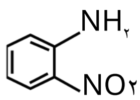
(C) 

(D) 

Choose the correct answer from the options given below : (١) (B) and (D) only (٣) (A) and (B) only

(٢) (A) and (C) only (٤) (A)، (B) and (C) only

Ans. (٣)

Sol. (A)  & (B) 

are steam volatile due to intramolecular hydrogen bonding

٧٠. Given below are two statements :

Statement (I) : The radii of isoelectronic species increases in the order.

$Mg^{٢+} < Na^{+} < F^{-} < O^{٢-}$

Statement (II) : The magnitude of electron gain enthalpy of halogen decreases in the order.

Cl < F < Br < I

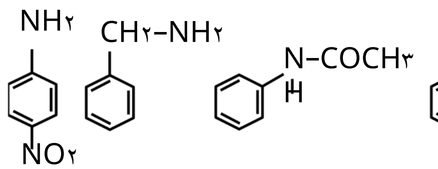
In the light of the above statements، choose the most appropriate answer from the options given below : (١) Statement I is incorrect but Statement II is correct (٢) Both Statement I and Statement II are incorrect (٣) Statement I is correct but Statement II is incorrect (٤) Both Statement I and Statement II are correct

Ans. (٤)

Sol. (i) For isoelectronic species –ve charge increases، radii increases. (ii) Magnitude of E.G.E : Cl < F < Br < I

SECTION-B

٧١. Given below are some nitrogen containing compounds.

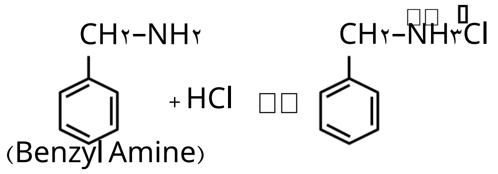


Each of them is treated with HCl separately. ١.٠ g of the most basic compound will consume ________mg of HCl.

(Given molar mass in g mol⁻¹ C : ١٢، H : ١، O : ١٦، Cl : ٣٥.٥)

Ans. (٣٤١)

Sol. Benzyl Amine is most basic due to localised lone pair.



Mole of benzyl Amine = $\frac{١}{١٠٧}$ = ٠.٠٠٩٣٤ mole

١ Mole of Benzyl amine consumed ١ mole of HCl

So، Mole of HCl consumed = ٠.٠٠٩٣٤ mole

Mass of HCl consumed = ٠.٠٠٩٣٤ × molar mass of HCl

= ٠.٠٠٩٣٤ × ٣٦.٥

= ٠.٣٤١ gm

= ٣٤١ mg

٧٢. The molar mass of the water insoluble product formed from the fusion of chromite ore (FeCr٢O٤) with Na٢CO٣ in presence of O٢ is ________ g mol⁻¹.

Ans. (١٦٠)

Sol. ٤FeCr٢O٤ + ٨Na٢CO٣ + ٧O٢ → ٨Na٢CrO٤ + ٢Fe٢O٣ + ٨CO٢

Fe٢O٣ is water insoluble، so its molar mass

= (٢ × ٥٦ + ٣ × ١٦) = ١٦٠ g/mol

٧٣. The sum of sigma (σ) and pi (π) bonds in Hex-١,٣-dien-٥-yne is __________.

Ans. (١٥)

Sol. H–C(H)=C(H)–C(H)=C(H)–C≡C–H

Number of σ bond = ١١

Number of π bond = ٤

σ + π = ١١ + ٤ = ١٥

٧٤. If A٢B is ٣٠% ionised in an aqueous solution، then the value of van't Hoff factor (i) is ______ × ١٠⁻¹.

Ans. (١٦)

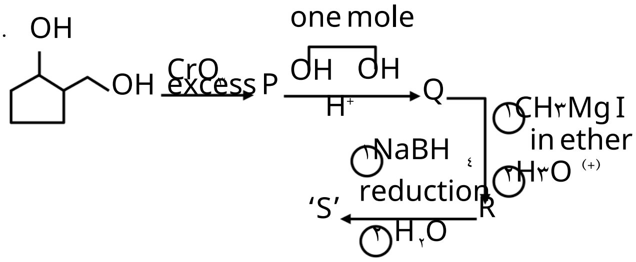
Sol. A٢B ⇌ ٢A + B ; y = ٣

α = ٠.٣

i = ١ + (y – ١)α

= ١ + (٣ – ١)(٠.٣) = ١.٦ = ١٦ × ١٠⁻¹

٧٥.

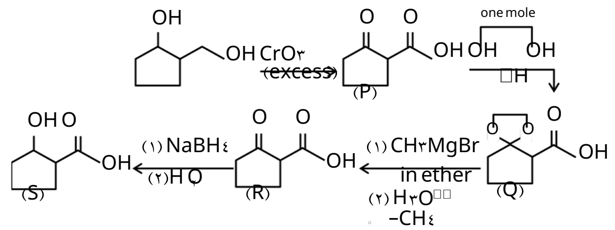


٠.١ mole of compound 'S' will weigh ______g.

(Given molar mass in g mol⁻¹ C : ١٢، H : ١، O : ١٦)

Ans. (١٣)

Sol.



٠.١ mole of compound (S) weight in gm

= ٠.١ × molar mass of compound (S)

= ٠.١ × ١٣٠ = ١٣ gm