JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON TUESDAY 28th JANUARY 2025) TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

١. The number of different ٥ digit numbers greater than ٥٠٠٠٠ that can be formed using the digits ٠، ١، ٢، ٣، ٤، ٥، ٦، ٧، such that the sum of their first and last digits should not be more than ٨، is

(١) ٤٦٠٨ (٢) ٥٧٢٠

(٣) ٥٧١٩ (٤) ٤٦٠٧

Ans. (٤)

Sol. Case I ٥ _ _ _ ٠

Case II ٥ _ _ _ ١

٥ ٢

٥ ٣

٦ ٠

٦ ١

٦ ٢

٧ ٠

Case IX ٧ _ _ _ ١

٩×(٨×٨×٨) = ٤٦٠٨ but ٥٠٠٠٠ is not included.

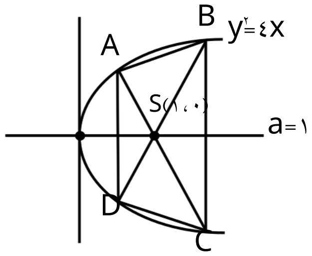
so total numbers ٤٦٠٨ − ١ = ٤٦٠٧

٢. Let ABCD be a trapezium whose vertices lie on the parabola $y^2 = 4x$. Let the sides AD and BC of the trapezium be parallel to y-axis. If the diagonal AC is of length $\frac{25}{4}$ and it passes through the point (١، ٠)، then the area of ABCD is :

(١) $\frac{75}{4}$ (٢) $\frac{25}{2}$

(٣) $\frac{125}{8}$ (٤) $\frac{75}{8}$

Ans. (١)

Sol. 

$A(at_1^2, 2at_1)$ & $C\left(\frac{a}{t_1^2}, \frac{-2a}{t_1}\right)$

Length $AC = a\left(t_1 + \frac{1}{t_1}\right)^2 = \frac{25}{4}$, $t_1 + \frac{1}{t_1} = \frac{5}{2}$

$\Rightarrow t_1 = 2$ or $\frac{1}{2}$, $A\left(\frac{1}{4}, 1\right)$, $D\left(\frac{1}{4}, -1\right)$, B(٤، ٤)، C(٤، −٤)

So, area of trapezium $= \frac{1}{2}(8 + 2)\left(4 - \frac{1}{4}\right) = \frac{75}{4}$

٣. Two number k_1 and k_2 are randomly chosen from the set of natural numbers. Then, the probability that the value of $i^{k_1} + i^{k_2}$, $(i = \sqrt{-1})$ is non-zero, equals

(١) $\frac{1}{2}$ (٢) $\frac{1}{4}$

(٣) $\frac{3}{4}$ (٤) $\frac{2}{3}$

Ans. (٣)

Sol. $i^{k_1} + i^{k_2} \neq 0$ $i^{k} \Rightarrow$ ٤ option for i، −١، −i، ١

Total cases = ٤×٤ = ١٦

Unfovourble cases $i^{k_1} + i^{k_2} = 0$

(1, −1)

(−1, 1)

(i, −i)

(−i, i)

٤ cases ⇒ Probability $= \frac{16 - 4}{16} = \frac{3}{4}$

٤. If $f(x) = \frac{2^x}{2^x + \sqrt{2}}$, $x \in R$, then $\sum_{k=1}^{81} f\left(\frac{k}{82}\right)$ is equal to :

(١) ٤١ (٢) $\frac{81}{2}$

(٣) ٨٢ (٤) $81\sqrt{2}$

Ans. (٢)

Sol. $f(x) = \frac{2^x}{2^x + \sqrt{2}}$

$f(x) + f(1-x) = \frac{2^x}{2^x + \sqrt{2}} + \frac{2^{1-x}}{2^{1-x} + \sqrt{2}}$

$= \frac{2^x}{2^x + \sqrt{2}} + \frac{2}{2 + \sqrt{2}\,2^x} = \frac{2^x}{2^x + \sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2} + 2^x} = 1$

Now, $\sum_{k=1}^{81} f\left(\frac{k}{82}\right) = f\left(\frac{1}{82}\right) + f\left(\frac{2}{82}\right) + \ldots\ldots + f\left(\frac{81}{82}\right)$

$= \left[f\left(\frac{1}{82}\right) + f\left(1 - \frac{1}{82}\right)\right] + \left[f\left(\frac{2}{82}\right) + f\left(1 - \frac{2}{82}\right)\right] + \ldots 40 \text{ cases} + f\left(\frac{41}{82}\right)$

$= (1 + 1 + \ldots 40 \text{ times}) + \frac{2^{1/2}}{2^{1/2} + 2^{1/2}}$

81

$40 + \frac{1}{2} = \frac{81}{2}$

٥. Let $f: R \to R$ be a function defined by

$f(x) = (٢ + ٣a)x^٢ + \left(\frac{a+٢}{a-١}\right)x + b$, $a \neq ١$. If

$f(x+y) = f(x) + f(y) + ١ - \frac{٢}{٧}xy$, then the value of

$٢٨\sum_{i=١}^{٥} |f(i)|$ is:

(١) ٧١٥ (٢) ٧٣٥

(٣) ٥٤٥ (٤) ٦٧٥

Ans. (٤)

Sol. $f(x) = (٣a + ٢)x^٢ + \left(\frac{a+2}{a-1}\right)x + b$

$f(x+y) = f(x) + f(y) + 1 - \frac{2}{7}xy$ ......(١)

In (١) Put $x = y = ٠ \Rightarrow f(٠) = ٢f(٠) + ١ \Rightarrow f(٠) = -١$

So, $f(٠) = ٠ + ٠ + b = -١ \Rightarrow b = -١$

In (١) Put $y = -x \Rightarrow f(٠) = f(x) + f(-x) + ١ + \frac{2}{7}x^2$

$-١ = ٢(٣a + ٢)x^٢ + ٢b + ١ + \frac{2}{7}x^2$

$-١ = \left(2(3a+2) + \frac{2}{7}\right)x^2 - 1$

$\Rightarrow ٦a + ٤ + \frac{2}{7} = ٠$

$a = -\frac{5}{7}$

So $f(x) = -\frac{1}{7}x^2 - \frac{3}{4}x - 1$

$\Rightarrow |f(x)| = \frac{1}{28}|4x^2 + 21x + 28|$

Now, $28\sum_{i=1}^{5}|f(i)| = 28(|f(1)| + |f(2)| + \ldots + |f(5)|)$

$28 \cdot \frac{1}{28} \cdot 675 = 675$

٦. Let A(x, y, z) be a point in xy-plane, which is equidistant from three points (٠, ٣, ٢), (٢, ٠, ٣) and (٠, ٠, ١).

Let B = (١, ٤, −١) and C = (٢, ٠, −٢). Then among the statements

(S١) : △ABC is an isosceles right angled triangle

and

(S٢) : the area of △ABC is $\frac{٩\sqrt{٢}}{٢}$.

(١) both are true (٢) only (S١) is true

(٣) only (S٢) is true (٤) both are false

Ans. (٢)

Sol. A(x, y, z) Let P(٠, ٣, ٢), Q(٢, ٠, ٣), R(٠, ٠, ١)

AP = AQ = AR

$x^٢ + (y-٣)^٢ + (z-٢)^٢ = (x-٢)^٢ + y^٢ + (z-٣)^٢ = x^٢ + y^٢ + (z-١)^٢$

In xy plane z = ٠

So, $x^٢ - ٤x + ٤ + y^٢ + ٩ = x^٢ + y^٢ + ١$

$\Rightarrow y = ٢$

$x = ٣$

$٩ + y^٢ - ٦y + ٩ + ٤ = x^٢ + y^٢ + ١$

So, A(٣, ٢, ٠) also B(١, ٤, −١) & C(٢, ٠, −٢)

Now AB = $\sqrt{4 + 4 + 1} = 3$

AC = $\sqrt{1 4 4 3}$

BC = $\sqrt{1 16 1}$ $\sqrt{18}$

AB = AC

isosceles & AB + AC = BC ٢

right angle

Area of ABC = $\frac{1}{2}$ × base . height

$\frac{1}{2}$ 3 3 $\frac{9}{2}$

So only S₁ is true

٧. The relation R = {(x, y) : x, y ∈ z and x + y is even} is :

(١) reflexive and transitive but not symmetric

(٢) reflexive and symmetric but not transitive

(٣) an equivalence relation

(٤) symmetric and transitive but not reflexive

Ans. (٣)

Sol. R = {(x, y) : x, y ∈ z and x + y is even}

reflexive x + x = ٢x even

symmetric of x + y is even, then (y + x) is also even

transitive of x + y is even & y + z is even then x + z is also even

So, relation is an equivalence relation.

٨. Let the equation of the circle, which touches x-axis at the point (a, ٠), a < ٠ and cuts off an intercept of length b on y-axis be x² + y² – αx + βy + γ = ٠. If the circle lies below x-axis, then the ordered pair (٢a, b²) is equal to :

(١)(α, β + ٤γ)

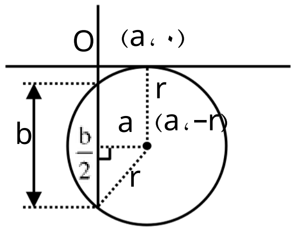
(٢)(α, β² – ٤γ)

(٣)(α, β² + ٤γ)

(٤)(α, β² – ٤γ)

Ans. (٤)

Sol.



By pytogorus $r^2 = a^2 + \frac{b^2}{4} = P^2$

$r = \sqrt{\frac{4a^2 + b^2}{4}}$

Equation of circle is (x – α)² + (y – β)² = r²

x² + y² – ٢ax – ٢py + α² + p² – r² = ٠

comparision x² + y² – αx + βy + r = ٠

–α = –٢a, β = –٢p, r = a²

α٢a = α, ٤a² + b² = ٤p²

α² + b² = ٤p²

α² + b² = β²

So, (٢a, b²) = (α, β² – ٤r)

٩. Let ⟨aₙ⟩ be a sequence such that a₀ = ٠, a₁ = $\frac{١}{٢}$ and

٢aₙ₊₂ = ٥aₙ₊₁ – ٣aₙ, n = ٠, ١, ٢, ٣, ... Then $\sum_{k=١}^{١٠٠} a_k$ is equal to :

(١) ٣a₉₉ – ١٠٠

(٢) ٣a₁₀₀ – ١٠٠

(٣) ٣a₁₀₀ + ١٠٠

(٤) ٣a₉₉ + ١٠٠

Ans. (٢)

Sol. a₀ = ٠, a₁ = $\frac{1}{2}$

٢aₙ₊₂ = ٥aₙ₊₁ – ٣aₙ

٢x² – ٥x + ٣ = ٠ ⇒ x = ١, ٣/٢

$a_n = A(١)^n + B\left(\frac{3}{2}\right)^n$

n = ٠ 0 = A + B A = –1

n = ١ $\frac{١}{٢}$ = A + $\frac{٣}{٢}$B B = ١

$a_n = -١ + \left(\frac{٣}{٢}\right)^n$

$\sum_{k=1}^{100} a_k = \sum_{k=1}^{100}(-1) + \left(\frac{3}{2}\right)^k$

$$= 100 + \frac{3\left(\left(\frac{3}{2}\right)^{100} - 1\right)}{\frac{3}{2} - 1}$$

$$= 100 + 3\left(\left(\frac{3}{2}\right)^{100} - 1\right)$$

= ٣.(a١٠٠) − ١٠٠

١٠. $\cos\left(\sin^{-1}\frac{٣}{٥} + \sin^{-1}\frac{٥}{١٣} + \sin^{-1}\frac{٣٣}{٦٥}\right)$ is equal to :

(١) ١ (٢) ٠

(٣) $\frac{٣٣}{٦٥}$ (٤) $\frac{٣٢}{٦٥}$

Ans. (٢)

Sol. $\cos\left(\sin^{-1}\frac{3}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{33}{65}\right)$

$\cos\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{5}{12} + \tan^{-1}\frac{33}{56}\right)$

$\cos\left(\tan^{-1}\left(\frac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4}\cdot\frac{5}{12}}\right) + \tan^{-1}\frac{33}{56}\right)$

$\cos\left(\tan^{-1}\frac{56}{33} + \cot^{-1}\frac{56}{33}\right)$

$\cos\left(\frac{\pi}{2}\right) = 0$

١١. Let T_r be the r^{th} term of an A.P. If for some m, $T_m = \frac{١}{٢٥}$, $T_{٢٥} = \frac{١}{٢٠}$ and $٢٠\sum_{r=1}^{٢٥} T_r = ١٣$, then

$٥m\sum_{r=m}^{٢m} T_r$ is equal to :

(١) ١١٢ (٢) ١٢٦

(٣) ٩٨ (٤) ١٤٢

Ans. (٢)

Sol. $T_m = \frac{١}{٢٥}$, $T_{٢٥} = \frac{١}{٢٠}$, $٢٠\sum_{r=1}^{٢٥} T_r = ١٣$

$T_m = a + (m - ١)d = \frac{1}{25}$ ......(١)

$T_{٢٥} = a + ٢٤d = \frac{1}{20}$

$20.\frac{25}{2}\left(a + \frac{1}{20}\right) = 13 \Rightarrow a = \frac{1}{500}$

also, $٢٠ S_{٢٥} = 20.\frac{25}{2}(2a + 24d) = 13 \Rightarrow d = \frac{1}{500}$

from (١) $\frac{1}{500} + \frac{m-1}{500} = \frac{1}{25} \Rightarrow m = 20$

Now,

$٥m\sum_{r=m}^{٢m} T_r = ١٠٠\sum_{r=٢٠}^{٤٠} T_r = ١٢٦$

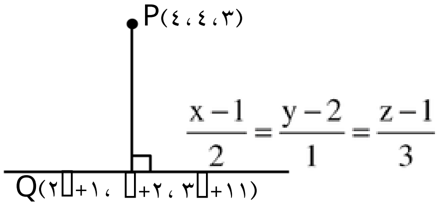
١٢. If the image of the point (٤, ٤, ٣) in the line $\frac{x-١}{٢} = \frac{y-٢}{١} = \frac{z-١}{٣}$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to

(١) ٩ (٢) ١٢

(٣) ٨ (٤) ٧

Ans. (١)

Sol.



$\overrightarrow{PQ} = (٢\lambda - ٣)\hat{i} + (\lambda - ٢)\hat{j} + (٣\lambda - ٢)\hat{k}$

$٢(٢\lambda - ٣) + ١(\lambda - ٢) + ٣(٣\lambda - ٢) = ٠$

$١٤\lambda - ١٤ = ٠$, $\lambda = ١$

So, Q(٣, ٣, ٤)

Let image in R(α, β, γ)

$\frac{\alpha + ٤}{٢} = ٣$, $\frac{\beta + ٤}{٢} = ٣$, $\frac{\gamma + ٣}{٢} = ٤$

(α, β, γ) = (٢, ٢, ٥)

α + β + γ = ٩

١٣. If $\int_{-\pi/٢}^{\pi/٢} \frac{٩٦x^٢\cos^٢ x}{(١ + e^x)}dx = \pi(\alpha\pi^٢ + \beta)$, α, β ∈ Z, then $(\alpha + \beta)^2$ equals :

(١) ١٤٤ (٢) ١٩٦

(٣) ١٠٠ (٤) ٦٤

Ans. (٣)

Sol. $\int_{-\pi/٢}^{\pi/٢} \frac{٩٦x^٢\cos^٢ x}{(١ + e^x)}dx$ (Apply King Property)

$\int_0^{\pi/2} 96x^2\cos^2 x = 48\int_0^{\pi/2} x^2(1+\cos 2x)\,dx$

$48\left[\left(\frac{x^3}{3}\right)_0^{\pi/2} + \int_0^{\pi/2} x^2\cos 2x\,dx\right]$

$\Rightarrow$ On solving $\pi(2\pi^2 - 12)$

$\therefore \alpha = 2,\ \beta = -12$

$\beta(\alpha+\beta) = 120$

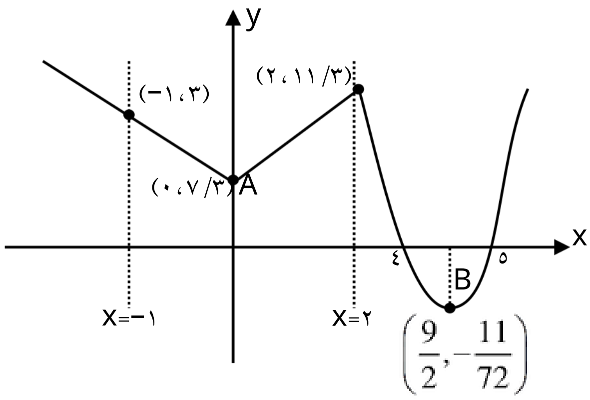
14. The sum of all local minimum values of the function $f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7+2|x|), & -1 \le x \le 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$

(1) $\frac{171}{72}$ (2) $\frac{131}{72}$

(3) $\frac{157}{72}$ (4) $\frac{167}{72}$

Ans. (3)

Sol. $f(x) = \begin{cases} 1-2x, & x < -1 \\ \frac{1}{3}(7-2x), & -1 \le x < 0 \\ \frac{1}{3}(7+2x), & 0 \le x \le 2 \\ \frac{11}{18}(x-4)(x-5), & x > 2 \end{cases}$



$\therefore$ Local minimum values at A & B

$\frac{7}{3} - \frac{11}{72}$

$= \frac{168 - 11}{72} = \frac{157}{72}$

15. The sum, of the squares of all the roots of the equation $x^2 + |2x - 3| - 4 = 0$, is :

(1) $3(3 - \sqrt{2})$ (2) $6(3 - \sqrt{2})$

(3) $6(2 - \sqrt{2})$ (4) $3(2 - \sqrt{2})$

Ans. (3)

Sol. $x^2 + |2x - 3| - 4 = 0$

Case I : $x \ge \frac{3}{2}$

$x^2 + 2x - 3 - 4 = 0$

$x^2 + 2x - 7 = 0$

$x = 2\sqrt{2} - 1$

Case II : $x < \frac{3}{2}$

$x^2 + 3 - 2x - 4 = 0$

$x^2 - 2x - 1 = 0$

$x = 1 - \sqrt{2}$

Sum of squares $= (2\sqrt{2} - 1)^2 + (1 - \sqrt{2})^2$

$= 8 - 4\sqrt{2} + 1 + 1 - 2\sqrt{2} + 2$

$= 6(2 - \sqrt{2})$ $\Rightarrow$ (3)

16. Let for some function $y = f(x)$, $\int_0^x t\,f(t)\,dt = x^2 f(x)$, $x > 0$ and $f(2) = 3$. Then $f(6)$ is equal to :

(1) 1 (2) 2

(3) 6 (4) 3

Ans. (1)

Sol. $\int_0^x t f(t)\,dt = x^2 f(x),\ x > 0$

Diff. both side w.r. to x

$xf(x) = x^2 f'(x) + 2xf(x)$

$-xf(x) = x^2 f'(x)$

$\int \frac{f'(x)}{f(x)}dx = \int \frac{-1}{x}dx$

$\log f(x) = -\log x + \log c$

$f(x) = \frac{c}{x}$

$f(2) = 3 \Rightarrow 3 = \frac{c}{2} \Rightarrow c = 6$

$f(x) = \frac{6}{x}$

$f(6) = 1$ $\Rightarrow$ (1)

١٧. Let $^{n}C_{r-١} = ٢٨$، $^{n}C_{r} = ٥٦$ and $^{n}C_{r+١} = ٧٠$. Let A(٤cost، ٤sint)، B(٢sint، −٢cost) and C(٣r − n، r−n−١) be the vertices of a triangle ABC، where t is a parameter. If $(٣x − ١)^٢ + (٣y)^٢ = \alpha$، is the locus of the centroid of triangle ABC، then α equals :

(١) ٢٠　　(٢) ٨

(٣) ٦　　(٤) ١٨

Ans.(١)

Sol. $^{n}C_{r-١} = ٢٨$، $^{n}C_{r} = ٥٦$

$$\frac{^{n}C_{r-١}}{^{n}C_{r}} = \frac{٢٨}{٥٦}$$

$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{١}{٢}$$

$$\frac{r}{(n-r+1)} = \frac{١}{٢}$$

٣r = n+١ —(i)

$$\frac{^{n}C_{r}}{^{n}C_{r+١}} = \frac{٥٦}{٧٠}$$

$$\frac{(r+١)}{(n-r)} = \frac{٥٦}{٧٠} \Rightarrow ٩r = ٤n−٥$$ —(ii)

By (i) & (ii)

(r = ٣)، (n = ٨)

A(٤cost، ٤sint) B(٢sint، −٢cost) C(٣r−n، r−n−١)

A(٤cost، ٤sint) B(٢sint، −٢cost) C(١، ٠)

$(٣x−١)^٢ + (٣y)^٢ = (٤cost + ٢sint)^٢ + (٤sint − ٢cost)^٢$

$(٣x − ١)^٢ + (٣y)^٢ = ٢٠$　$\Rightarrow$ option (١)

١٨. Let O be the origin، the point A be $z_١ = \sqrt{٣} + ٢\sqrt{٢}i$، the point B($z_٢$) be such that $\sqrt{٣}|z_٢| = |z_١|$ and $\arg(z_٢) = \arg(z_١) + \frac{\pi}{6}$. Then

(١) area of triangle ABO is $\frac{١١}{\sqrt{٣}}$

(٢) ABO is a scalene triangle

(٣) area of triangle ABO is $\frac{١١}{٤}$

(٤) ABO is an obtuse angled isosceles triangle

Ans. (٤)

Sol. $z_١ = \sqrt{٣} + ٢\sqrt{٢}i$ & $\frac{|z_٢|}{|z_١|} = \frac{١}{\sqrt{٣}}$

given $\arg\left(\frac{z_٢}{z_١}\right) = \frac{\pi}{٦}$

$$z_٢ = \frac{|z_٢|}{|z_١|} \cdot z_١ e^{i\frac{\pi}{6}}$$

$$z_٢ = \frac{١}{\sqrt{٣}} \cdot \frac{(\sqrt{٣} + ٢\sqrt{٢}i)(\sqrt{٣} + i)}{٢}$$

$$z_٢ = \frac{(٣ - ٢\sqrt{٢}) + i(\sqrt{٣} + ٢\sqrt{٦})}{٢\sqrt{٣}}$$

Now،

$$z_١ - z_٢ = \frac{(3 + \sqrt{3}) + i(\sqrt{} + ٦\sqrt{٣})}{٢\sqrt{٣}}$$

$|z_1-z_2| = |z_2| \Rightarrow \Delta$ ABO is isosceles with angles $\frac{\pi}{٦}, \frac{\pi}{٦}$ & $\frac{٢\pi}{٣}$

١٩. Three defective oranges are accidently mixed with seven good ones and on looking at them، it is not possible to differentiate between them. Two oranges are drawn at random from the lot. If x denote the number of defective oranges، then the variance of x is :

(١) ٢٨/٧٥　　(٢) ١٤/٢٥

(٣) ٢٦/٧٥　　(٤) ١٨/٢٥

Ans. (١)

Sol. ١٠ oranges → ٧ good, ٣ defected

Probability distribution

x_i	p_i
$x = ٠$	$\frac{^{٧}C_٢}{^{١٠}C_٢} = \frac{٤٢}{٩٠}$
$x = ١$	$\frac{^{٧}C_١ \times {^{٣}C_١}}{^{١٠}C_٢} = \frac{٤٢}{٩٠}$
$x = ٢$	$\frac{^{٣}C_٢}{^{١٠}C_٢} = \frac{٦}{٩٠}$

Now،

$$\mu = \sum x_i p_i = \frac{٤٢}{٩٠} + \frac{١٢}{٩٠} = \frac{٥٤}{٩٠}$$

$$\sigma^٢ = \sum p_i x_i^٢ - \mu^٢ = \frac{٤٢}{٩٠} + \frac{٢٤}{٩٠} - \left(\frac{٥٤}{٩٠}\right)^٢$$

$$= \frac{٦٦}{٩٠} - \left(\frac{٥٤}{٩٠}\right)^٢$$

$$\sigma^٢ = \frac{٢٨}{٧٥} \Rightarrow (١)$$

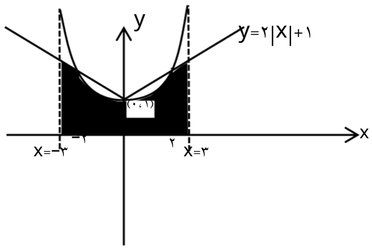
٢٠. The area (in sq. units) of the region $\{(x, y) : 0 \le y \le 2|x| + 1, 0 \le y \le x^2 + 1, |x| \le 3\}$ is

(١) $\frac{٨٠}{٣}$ (٢) $\frac{٦٤}{٣}$

(٣) $\frac{١٧}{٣}$ (٤) $\frac{٣٢}{٣}$

Ans. (٢)

Sol.



Area = ٢ [illegible] $\int_0^2 (x^2+1)dx + \int_2^3 (2x+1)dx$

= $\frac{٦٤}{٣}$ ⟹ (٢)

SECTION-B

٢١. Let M denote the set of all real matrices of order ٣ × ٣ and let S = {−٣, −٢, −١, ١, ٢}. Let

$S_1 = \{A = [a_{ij}] \in M : A = A^T$ and $a_{ij} \in S, \forall i, j\}$

$S_2 = \{A = [a_{ij}] \in M : A = -A^T$ and $a_{ij} \in S, \forall i, j\}$

$S_3 = \{A = [a_{ij}] \in M : a_{11} + a_{22} + a_{33} = 0$ and $a_{ij} \in S, \forall i, j\}$

If $n(S_1 \cup S_2 \cup S_3) = ١٢٥\alpha$, then α equals.

Ans. (١٦١٣)

Sol. $\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

No. of elements in S_1 : $A = A^T \Rightarrow ٥^٦$

No. of elements in S_2 : $A = -A^T \Rightarrow ٠$

since no zero in S

No. of elements in S_3

$a_{11}+a_{22}+a_{33} = 0$ [illegible] (١، ٢، −٣) → ٣!

or

(١، ١، −٢) → ٣

or

(−١، −١، ٢) → ٣

⟹ ١٢ × ٥^٦

$n(S_1 \cap S_3) = ١٢ × ٥^٣$

$n(S_1 \cup S_2 \cup S_3) = ٥^٣(١+١٢) − ١٢ × ٥^٣$

⟹ $٥^٣ × ١٣ × ٥^٣ − ١٢ × ٥^٣ = ١٢٥\alpha$

⟹ α = ١٦١٣

٢٢. If $\alpha = 1 + \sum_{r=1}^{6} (-3)^{r-1}\, {}^{12}C_{2r-1}$, then the distance of the point (١٢, $\sqrt{3}$) form the line $\alpha x - \sqrt{3}y + 1 = 0$ is ... ...

Ans. (٥)

Sol. $\alpha = 1 + \sum_{r=1}^{6} (-1)^{r-1}\, {}^{12}C_{2r-1}\, 3^{r-1}$

$\alpha = 1 + \sum_{r=1}^{6} {}^{12}C_{2r-1} \frac{(\sqrt{3}i)^{2r-1}}{\sqrt{3}i}$ i = iota, let $\sqrt{3}i = x$

$\alpha = 1 + \frac{1}{\sqrt{3}i}\left[{}^{12}C_1 x + {}^{12}C_3 x^3 + \ldots + {}^{12}C_{11} x^{11}\right]$

$= 1 + \frac{1}{\sqrt{3}i}\left[\frac{(1+\sqrt{3}i)^{12} - (1-\sqrt{3}i)^{12}}{2}\right]$

$= 1 + \frac{1}{\sqrt{3}i}\left[\frac{2^{12} - 2^{12}}{2}\right] = 1$

so distance of (١٢, $\sqrt{3}$) from $x - \sqrt{3}y + 1 = 0$ is

$\frac{١٢ − ٣ + ١}{٢} = ٥$

٢٣. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{d} = \vec{a} \times \vec{b}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - 2\vec{a}|^2 = 8$ then and the angle between $\vec{d}$ and $\vec{c}$ is $\frac{\pi}{4}$, $|10 - 3\vec{b} \cdot \vec{c}| + |\vec{d} \times \vec{c}|^2$ is equal to ... . .

Ans. (٦)

Sol. $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

$\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$

$\vec{d} = \vec{a} \times \vec{b}$

$= -\hat{i} + \hat{j}$

$|\vec{c} - 2\vec{a}|^2 = 8$

$|\vec{c}|^2 + 4|\vec{a}|^2 - 4(\vec{a} \cdot \vec{c}) = 8$

$c^2 + 12 - 4c = 8$

$c^2 - 4c + 4 = 0$

$|\vec{c}| = 2$

$\vec{d} = \vec{a} \times \vec{b}$

$\vec{d} \times \vec{c} = (\vec{a} \times \vec{b}) \times \vec{c}$

$|\vec{d}||\vec{c}| \sin\frac{\pi}{4} = |(\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}|$

$4 = 4b^2 + (\vec{b} \cdot \vec{c})^2 (a)^2 - 4(\vec{b} \cdot \vec{c})(\vec{a} \cdot \vec{b})$

Let b . c = x

٤ = ٣٦ + ٣x − ٢٠x

٣x٢ − ٢٠x + ٣٢ = ٠

٣x٢ − ١٢x − ٨x + ٣٢ = ٠

x = ٨/٣ ، ٤

b . c = ٨/٣ ، ٤

b . c = ٨/٣

Now |١٠ − ٣b . c| + |d × c|٢

|١٠ − ٨| + (٢) ٢

= ٦ Ans.

٢٤. Let

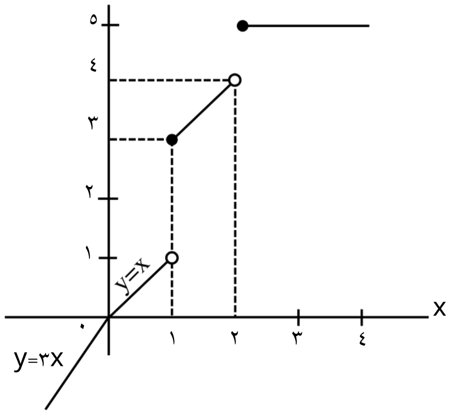
f(x) = { ٣x ، x < ٠ ; min{١ + x + [x], x + ٢[x]} ، ٠ ≤ x ≤ ٢ ; ٥ ، x > ٢ }

where [.] denotes greatest integer function. If α and β are the number of points, where f is not continuous and is not differentiable, respectively, then α + β equals ... .

Ans. (٥)

Sol. f(x) = { ٣x ، x < ٠ ; min{١ + x, x} ، ٠ ≤ x < ١ ; min{٢ + x, x + ٢} ، ١ ≤ x < ٢ ; ٥ ، x ≥ ٢ }

f(x) = { ٣x ، x < ٠ ; x ، ٠ ≤ x < ١ ; x + ٢ ، ١ ≤ x < ٢ ; ٥ ، x ≥ ٢ }



Not continuous at x = {١ ، ٢} ⇒ α = ٢

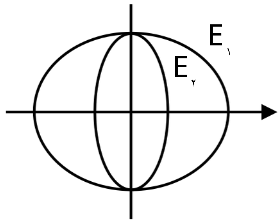
Not diff. at x = {٠ ، ١ ، ٢} ⇒ β = ٣

α + β = ٥

٢٥. Let $E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ be an ellipse. Ellipses E_i's are constructed such that their centres and eccentricities are same as that of E_1 and the length of minor axis of E_i is the length of major axis of E_{i+1} (i ≥ ١). If A_i is the area of the ellipse E_i, then $\frac{5}{\pi}\left(\sum_{i=1}^{\infty} A_i\right)$ is equal to ... ...

Ans. (٥٤)

Sol.



$E_1 : \frac{x^2}{9} + \frac{y^2}{4} = 1$ ⇒ e = $\sqrt{1 - \frac{4}{9}} = \frac{\sqrt{5}}{3}$

$E_2 : \frac{x^2}{a^2} + \frac{y^2}{4} = 1$

e = $\frac{\sqrt{5}}{3} = \sqrt{1 - \frac{a^2}{4}}$ ⇒ $\frac{5}{9} = 1 - \frac{a^2}{4}$

$a^2 = \frac{16}{9}$

$E_2 : \frac{x^2}{16/9} + \frac{y^2}{4} = 1$

$E_3 : \frac{x^2}{16/9} + \frac{y^2}{b^2} = 1$

e = $\frac{\sqrt{5}}{3} = \sqrt{1 - \frac{b^2}{16/9}}$ ⇒ $b^2 = \frac{64}{81}$

$E_3 = \frac{x^2}{16/9} + \frac{y^2}{64/81} = 1$

$A_1 = \pi \times 3 \times 2 = 6\pi$

$A_2 = \pi \cdot \frac{4}{3} \cdot 2 = \frac{8\pi}{3}$

$A_3 = \pi \cdot \frac{4}{3} \cdot \frac{8}{9} = \frac{32\pi}{81}$

$\sum_{i=1}^{\infty} A_i = 6\pi + \frac{8\pi}{3} + \frac{32\pi}{81} + \ldots = \frac{6\pi}{1 - \frac{4}{9}} = \frac{54\pi}{5}$

$\frac{5}{\pi}\sum_{i=1}^{\infty} A_i = \frac{5}{\pi} \cdot \frac{54\pi}{5} = 54$

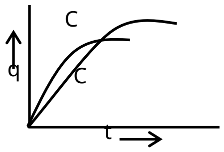
JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON TUESDAY28 th JANUARY2025) TIME : 9 : 00AM TO 12 : 00 NOON

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

٢٦. Two capacitors $C_١$ and $C_٢$ are connected in parallel to a battery. Charge-time graph is shown below for the two capacitors. The energy stored with them are $U_١$ and $U_٢$ respectively. Which of the given statements is true ؟



(١) $C_١ < C_٢$, $U_١ < U_٢$ (٢) $C_١ < C_٢$, $U_٢ > U_١$

(٣) $C_١ < C_٢$, $U_١ > U_٢$ (٤) $C_٢ < C_١$, $U_٢ < U_١$

Ans. (٤)

Sol. V same potential difference،

$U = \frac{١}{٢} cv^٢$

as $q_١ > q_٢$

$\therefore c_١ > c_٢$

& $U_١ > U_٢$

٢٧. In the experiment for measurement of viscosity 'η' of given liquid with a ball having radius R، consider following statements.

A. Graph between terminal velocity V and R will be a parabola

B. The terminal velocities of different diameter balls are constant for a given liquid.

C. Measurement of terminal velocity is dependent on the temperature.

D. This experiment can be utilized to assess the density of a given liquid.

E. If balls are dropped with some initial speed، the value of η will change.

Choose the correct answer from the options given below:

(١) B، D and E only

(٢) A، C and D only

(٣) C، D and E only

(٤) A، B and E only

Ans. (٢)

Sol. $V_T = \frac{٢}{٩} R^٢ \frac{g}{\eta}(\rho - \sigma)$

٢٨. Consider following statements:

A. Surface tension arises due to extra energy of the molecules at the interior as compared to the molecules at the surface، of a liquid.

B. As the temperature of liquid rises، the coefficient of viscosity increases.

C. As the temperature of gas increases، the coefficient of viscosity increases.

D. The onset of turbulence is determined by Reynold's number.

E. In a steady flow two stream lines never intersect.

Choose the correct answer from the options given below:

(١) A، D، E only

(٢) C، D، E only

(٣) B، C، D only

(٤) A، B، C only

Ans. (٢)

٢٩. Three infinitely long wires with linear charge density λ are placed along the x-axis، y-axis and z-axis respectively. Which of the following denotes an equipotential surface ؟

(١) $xy + yz + zx =$ constant

(٢) $(x + y)(y + z)(z + x) =$ constant

(٣) $(x^٢ + y^٢)(y^٢ + z^٢)(z^٢ + x^٢) =$ constant

(٤) $xyz =$ constant

Ans. (٣)

Sol. $v = -\int \vec{E}\cdot d\vec{r} = -\int \frac{2k\lambda}{r}dr = -2k\lambda \ln r + c$

Net potential due to all wire

$v = -2k\lambda \ln\sqrt{x^2+y^2} - 2k\lambda \ln\sqrt{y^2+z^2} - 2k\lambda \ln\sqrt{z^2+x^2} + c$

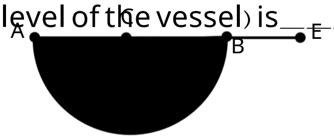
for v = c

$\sqrt{(x^2+y^2)(y^2+z^2)(z^2+x^2)} = C$

$\Rightarrow (x^2+y^2)(y^2+z^2)(z^2+x^2) = C$

where c = constant

٣٠. A hemispherical vessel is completely filled with a liquid of refractive index μ. A small coin is kept at the lowest point (O) of the vessel as shown in figure. The minimum value of the refractive index of the liquid so that a person can see the coin from point E (at the level of the vessel) is________.

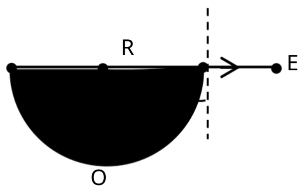


(١) $\sqrt{3}$ (٢) $\frac{3}{2}$

(٣) $\sqrt{2}$ (٤) $\frac{\sqrt{3}}{2}$

Ans. (٣)

Sol.



$\sin c = \frac{1}{\mu}$

for μ least, c maximum

$\theta = c = ٤٥$

$\Rightarrow \mu = \frac{1}{\sin ٤٥} = \sqrt{٢}$

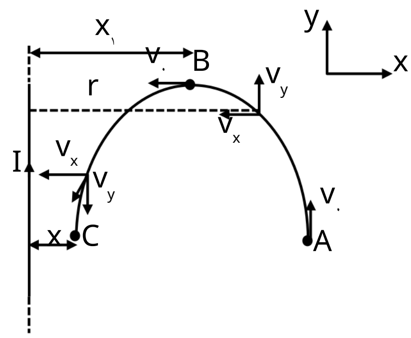
٣١. Consider a long thin conducting wire carrying a uniform current I. A particle having mass "M" and charge "q" is released at a distance "a" from the wire with a speed v_o along the direction of current in the wire. The particle gets attracted to the wire due to magnetic force. The particle turns round when it is at distance x from the wire. The value of x is (μ_o is vacuum permeability)

(١) $a\left[1 - \frac{mv_o}{2q\mu_o I}\right]$ (٢) $\frac{a}{2}$

(٣) $a\left[1 - \frac{mv_o}{q\mu_o I}\right]$ (٤) $ae^{\frac{-4\pi m v_o}{q\mu_o I}}$

Ans. (٤)

Sol.



A → B

$\vec{V} = -v_x\hat{i} + v_y\hat{j}$

$B = \frac{\mu_0 I}{2\pi r}(-\hat{k})$

$F = q(\vec{v}\times\vec{B}) = \frac{\mu_0 I q}{2\pi r}[-v_x\hat{j} - v_y\hat{i}]$

$a_x = -\frac{\mu_0 I q}{2\pi m}\cdot\frac{v_y}{r}$

$a_y = -\frac{\mu_0 I q}{2\pi m}\cdot\frac{v_x}{r}$

$\frac{v_x dv_x}{dr} = -\frac{\mu_0 I q}{2\pi m r}v_y$

$\frac{v_x dv_x}{v_y} = -\frac{\mu_0 I q}{2\pi m}\frac{dr}{r}$

$\int_0^{v_0}\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = -\frac{\mu_0 I q}{2\pi m}\int_a^x\frac{dr}{r}$

Let, $z = v_0^2 - v_x^2$

$2zdz = -2v_x dv_x$

$zdz = -v_x dv_x$

$$\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{-zdz}{z} = -dz$$

then integral becomes

$$-\int_{v_0}^{0} dz = -\frac{\mu_0 I q \ln}{2\pi m}\frac{x_1}{a}$$

$$v_0 = -\frac{\mu_0 I q}{2\pi m}\ln\frac{x_1}{a}$$

$$x_1 = a\,e^{-\frac{2\pi m v_0}{\mu_0 I q}} \quad \ldots\ldots(١)$$

For B → C

$$\vec{v} = -v_x\hat{i} - v_y\hat{j}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{k})$$

$$\vec{F} = q(\vec{v}\times\vec{B}) = \frac{\mu_0 I q}{2\pi r}(-v_x\hat{j} + v_y\hat{i})$$

$$a_x = +\frac{\mu_0 I q v_y}{2\pi m r} \qquad a_y = -\frac{\mu_0 I q}{2\pi m}\cdot\frac{v_x}{r}$$

$$\frac{v_x dv_x}{dr} = \frac{\mu_0 I q v_y}{2\pi m r}$$

$$\int_{v_0}^{0}\frac{v_x dv_x}{\sqrt{v_0^2 - v_x^2}} = \frac{\mu_0 I q}{2\pi m}\int_{x_1}^{x}\frac{dr}{r}$$

$$\frac{\mu_0 I q}{2\pi m}\ln\frac{x}{x_1} = -\int_{0}^{v_0} dz = -v_0$$

$$x = x_1 e^{-\frac{2\pi m v_0}{\mu_0 I q}} \quad \ldots(٢)$$

From equation ١ and ٢

$$X = a\,e^{-\frac{4\pi m v_0}{\mu_0 I q}}$$

٣٢. A Carnot engine (E) is working between two temperatures ٤٧٣K and ٢٧٣K. In a new system two engines – engine $E_١$ works between ٤٧٣K to ٣٧٣K and engine $E_٢$ works between ٣٧٣K to ٢٧٣K. If $\eta_{١٢}$, $\eta_١$ and $\eta_٢$ are the efficiencies of the engines E, $E_١$ and $E_٢$ respectively, then

(١) $\eta_{١٢} > \eta_١ + \eta_٢$ (٢) $\eta_{١٢} = \eta_١\eta_٢$

(٣) $\eta_{١٢} = \eta_١ + \eta_٢$ (٤) $\eta_{١٢} \geq \eta_١ + \eta_٢$

Ans. (١)

Sol. $\eta_{١٢} = ١ - \frac{٢٧٣}{٤٧٣} = \frac{٢٠٠}{٤٧٣} = ٠.٤٢٣$

$\eta_١ = 1 - \frac{٣٧٣}{٤٧٣} = \frac{١٠٠}{٤٧٣} = ٠.٢١١$

$\eta_٢ = 1 - \frac{٢٧٣}{٣٧٣} = \frac{١٠٠}{٣٧٣} = ٠.٢٦٨$

٣٣. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R Assertion A: A sound wave has higher speed in solids than gases.

Reason R: Gases have higher value of Bulk modulus than solids.

In the light of the above statements, choose the

correct answer from the options given below

(١) Both A and R are true and R is the correct explanation of A

(٢) A is false but R is true

(٣) Both A and R are true but R is NOT the correct

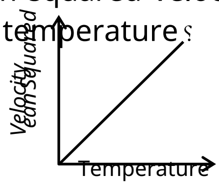
explanation of A

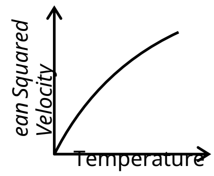
(٤) A is true but R is false.

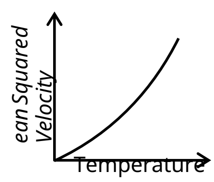
Ans. (٤)

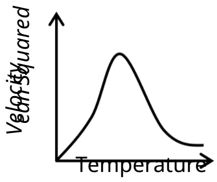
Sol. Solids have higher value of bulk modulus than gases.

٣٤. For a particular ideal gas which of the following graphs represents the variation of mean squared velocity of the gas molecules with temperature?

(١) 

(٢) 

(٣) 

(٤) 

Ans. (١)

Sol. $V_{rms} = \sqrt{\frac{3RT}{M}}$

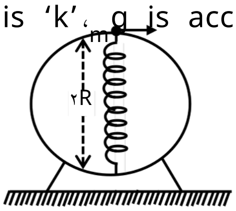
$V^2_{rms} = ٣RT/M$

Hence we can conclude that V^2_{rms} is directly proportional to temperature

y = m x

∴ Graph will be straight line.

٣٥. A bead of mass 'm' slides without friction on the wall of a vertical circular hoop of radius 'R' as shown in figure. The bead moves under the combined action of gravity and a massless spring (k) attached to the bottom of the hoop. The equilibrium length of the spring is 'R'. If the bead is released from top of the hoop with (negligible) zero initial speed, velocity of bead, when the length of spring becomes 'R', would be (spring constant is 'k', g is acceleration due to gravity)

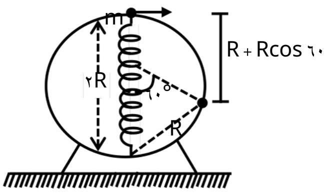


(١) $2\sqrt{gR + \frac{kR^2}{m}}$ (٢) $\sqrt{2Rg + \frac{4kR^2}{m}}$

(٣) $\sqrt{2Rg + \frac{kR^2}{m}}$ (٤) $\sqrt{3Rg + \frac{kR^2}{m}}$

Ans. (٤)

Sol.



Work energy theorem

Mg (R + Rcos٦٠) + $\frac{1}{2}k(R)^2 = \frac{1}{2}mv^2$

Mg $\frac{3R}{2} + \frac{KR^2}{2} = \frac{1}{2}mv^2$

V = $\sqrt{3gR + \frac{KR^2}{m}}$

٣٦. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R

Assertion A: In a central force field, the work done is independent of the path chosen

Reason R: Every force encountered in mechanics does not have an associated potential energy.

In the light of the above statements, choose the most appropriate answer from the options given below (١)A is true but R is false (٢)Both A and R are true but R is NOT the correct explanation of A (٣)Both A and R are true and R is the correct explanation of A (٤)A is false but R is true

Ans. (٢)

Sol. Both statement are correct but Reason is not the correct explanation of Assertion.

٣٧. Choose the correct nuclear process from the below options

[p: proton, n: neutron, e^-: electron, e^+: positron, ν: neutrino, $\bar{\nu}$: antineutrino]

(١) $n \to p + e^- + \bar{\nu}$ (٢) $n \to p + e^+ + \bar{\nu}$

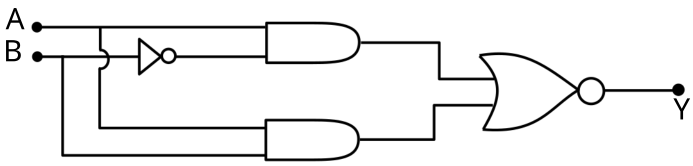
(٣) $n \to p + e^- + \nu$ (٤) $n \to p + e^+ + \nu$

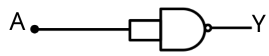
Ans. (١)

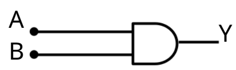
Sol. Theoretical equation for β^- decay

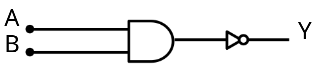
$$^1_0n \to {}^1_1p + {}^0_{-1}e + \bar{\nu}$$

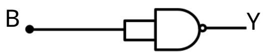
٣٨. Which of the following circuits has the same output as that of the given circuit?



(١)


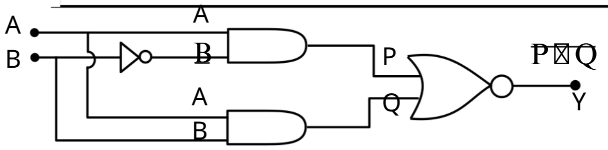
(٢)


(٣)


(٤)


Ans. (١)

Sol.



P = A⊕B

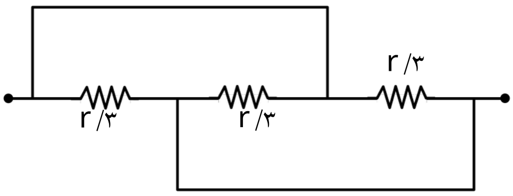
Q = A⊕B

$Y = \overline{P \oplus Q} = \overline{A \oplus B \oplus A \oplus B}$

$= \overline{A \oplus (B \oplus B) \oplus A \oplus 1}$

Y = $\overline{A}$

٣٩. Find the equivalent resistance between two ends of the following circuit.

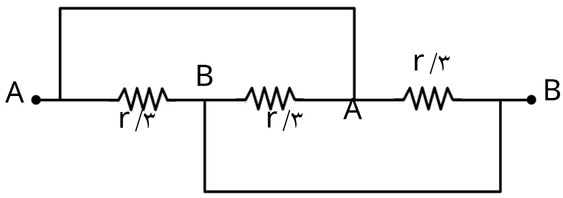


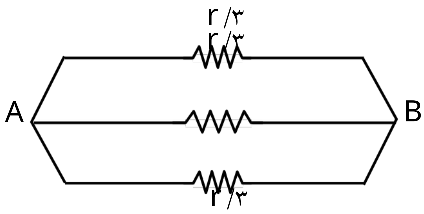
(١) r (٢) $\frac{r}{٦}$

(٣) $\frac{r}{٩}$ (٤) $\frac{r}{٣}$

Ans. (٣)

Sol.





All are in parallel

$R_{eq} = \frac{r/3}{3} = r/9$

٤٠. A wire of resistance R is bent into an equilateral triangle and an identical wire is bent into a square. The ratio of resistance between the two end points of an edge of the triangle to that of the square is

(١) ٩/٨ (٢) ٨/٩

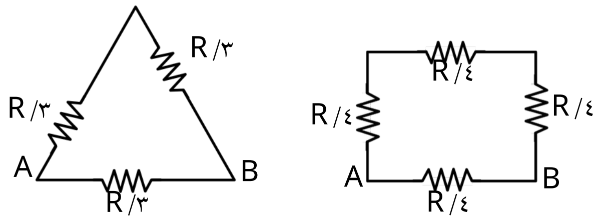
(٣) ٢٧/٣٢ (٤) ٣٢/٢٧

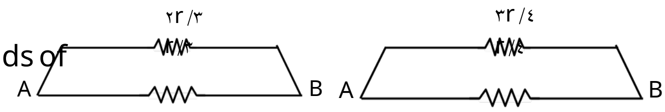
Ans. (٤)

Sol. $R = \frac{\rho\ell}{A}$

So, R ∝ ℓ

Side length of triangle is ١/٣ of total length.





$(R_{eq})_1 = \frac{2r/3 \times r/3}{2r/3 + r/3}$ $(R_{eq})_2 = \frac{3r/4 \times r/4}{3r/4 + r/4}$

$(R_{eq})_1 = ٢r/٩$ $(R_{eq})_2 = ٣r/١٦$

$\frac{(R_{eq})_1}{(R_{eq})_2} = \frac{2r/9}{3r/16} = \frac{32}{27}$

٤١. Due to presence of an em-wave whose electric component is given by E = ١٠٠ sin(ωt−kx) NC⁻¹, a cylinder of length ٢٠٠ cm holds certain amount of em-energy inside it. If another cylinder of same length but half diameter than previous one holds same amount of em-energy, the magnitude of the electric field of the corresponding em-wave should be modified as

(١) ٢٥ sin(ωt−kx) NC⁻¹

(٢) ٢٠٠ sin(ωt−kx) NC⁻¹

(٣) ٤٠٠ sin(ωt−kx) NC⁻¹

(٤) ٥٠ sin(ωt−kx) NC⁻¹

Ans. (٢)

Sol. Energy density = $\frac{1}{2}\varepsilon_0 E^2 c$

Energy = $\frac{1}{2}\varepsilon_0 E^2 \times c \times volume$

$(Energy)_1 = (Energy)_2$ (Given)

$\frac{1}{2}\varepsilon_0 E_1^2 c \pi R_1^2 L = \frac{1}{2}\varepsilon_0 E_2^2 c \pi R_2^2 L$

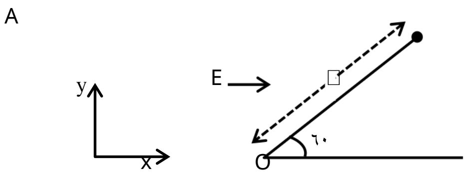
$E_1^2 R_1^2 = E_2^2 R_2^2$

$E_1 R_1 = E_2 R_2$

$١٠٠ \times R_1 = E_2 \times \frac{R_1}{2}$

$E_2 = ٢٠٠$ N/C

٤٢. A particle of mass 'm' and charge 'q' is fastened to one end 'A' of a massless string having equilibrium length ℓ, whose other end is fixed at point 'O'. The whole system is placed on a frictionless horizontal plane and is initially at rest. If uniform electric field is switched on along the direction as shown in figure, then the speed of the particle when it crosses the x-axis is

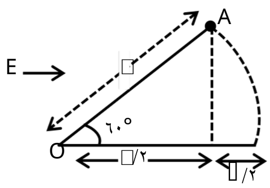


(١) $\sqrt{\frac{2qE\ell}{m}}$ (٢) $\sqrt{\frac{qE\ell}{4m}}$

(٣) $\sqrt{\frac{qE\ell}{m}}$ (٤) $\sqrt{\frac{qE\ell}{2m}}$

Ans. (٣)

Sol.



$W_{all} = \Delta k$

$W_e = k_f - k_i$

$qE\frac{\ell}{2} = \frac{1}{2}mv^2 - 0$

$v = \sqrt{\frac{qE\ell}{m}}$

٤٣. A proton of mass 'm_p' has same energy as that of a photon of wavelength 'λ'. If the proton is moving at non-relativistic speed, then ratio of its de Broglie wavelength to the wavelength of photon is.

(١) $\frac{1}{c}\sqrt{\frac{2E}{m_p}}$ (٢) $\frac{1}{c}\sqrt{\frac{E}{m_p}}$

(٣) $\frac{1}{c}\sqrt{\frac{E}{2m_p}}$ (٤) $\frac{1}{2c}\sqrt{\frac{E}{m_p}}$

Ans. (٣)

Sol. E is missing in the question but considering E as energy, the solution will be

$E_{photon} = \frac{hc}{\lambda} = E$; $E_{proton} = \frac{1}{2}m_p v^2 = E$

$\frac{\lambda_{proton}}{\lambda_{photon}} = \frac{h/p}{hc/E} = \frac{h/\sqrt{2m_pE}}{hc/E}$

$= \frac{E}{c\sqrt{2m_pE}}$

$\frac{\lambda_{proton}}{\lambda_{photon}} = \frac{1}{c}\sqrt{\frac{E}{2m_p}}$

٤٤. The centre of mass of a thin rectangular plate (fig - x) with sides of length a and b, whose mass per unit area (σ) varies as $\sigma = \frac{\sigma_0 x}{ab}$ (where σ_0 is a constant), would be________

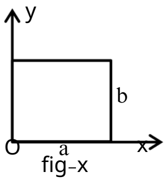


fig-x

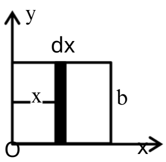
(١) $\left(\frac{2}{3}a, \frac{b}{2}\right)$ (٢) $\left(\frac{2}{3}a, \frac{2}{3}b\right)$

(٣) $\left(\frac{a}{2}, \frac{b}{2}\right)$ (٤) $\left(\frac{1}{3}a, \frac{b}{2}\right)$

Ans. (١)

Sol. σ is constant in y-direction

So, $y_{cm} = b/2$



$x_{cm} = \frac{\int_0^a x\,dm}{\int_0^a dm}$

$$= \frac{\int_0^a x \cdot x\,dA}{\int_0^a x\,dA}$$

$$= \frac{\int_0^a x\left(\frac{b}{a}x\right)b\,dx}{\int_0^a \left(\frac{b}{a}x\right)b\,dx}$$

$$x_{cm} = \frac{\int_0^a x^2dx}{\int_0^a x\,dx}$$

$$= \frac{\left[\frac{x^3}{3}\right]_0^a}{\left[\frac{x^2}{2}\right]_0^a} = \frac{a^3/3}{a^2/2}$$

$$= \frac{2a}{3}$$

٤٥. A thin prism P_1 with angle ٤° made of glass having refractive index ١.٥٤، is combined with another thin prism P_2 made of glass having refractive index ١.٧٢ to get dispersion without deviation. The angle of the prism P_2 in degrees is

(١) ٤ (٢) ٣

(٣) ١٦/٣ (٤) ١.٥

Ans. (٢)

Sol. $\delta_{net} = 0$

$(\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0$

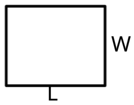
$(1.54 - 1)4 - (1.72 - 1)A_2 = 0$

A_2 = ٣°

٤٦. A tiny metallic rectangular sheet has length and breadth of ٥ mm and ٢.٥mm, respectively. Using a specially designed screw gauge which has pitch of ٠.٧٥ mm and ١٥ divisions in the circular scale, you are asked to find the area of the sheet. In this measurement, the maximum fractional error will be $\frac{x}{100}$ where x is________

Ans. (٣)

Sol.



Since least count of the instrument can be calculated as

$$\text{Least count} = \frac{\text{pitch length}}{\text{No.of division on circular scale}}$$

$$= \frac{0.75}{15} = 0.05\text{mm}.$$

Here we are provided L = ٥ mm & W = ٢.٥ mm

L = ٥ mm & W = ٢.٥ mm

∵ We know that

A = L.W

For calculating fractional error, we can write

$$\frac{dA}{A} = \frac{dL}{L} + \frac{dW}{W}$$

Here dL = dW = ٠.٠٥ mm

$$\frac{dA}{A} = \frac{0.05}{5} + \frac{0.05}{2.5}$$

$$\Rightarrow \frac{dA}{A} = \frac{1}{100} + \frac{2}{100} = \frac{3}{100},$$

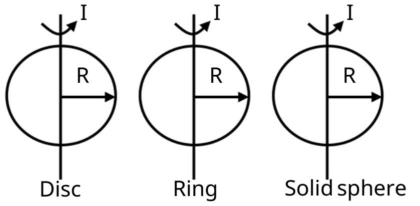
So, x = ٣

٤٧. The moment of inertia of a solid disc rotating along its diameter is ٢.٥ times higher than the moment of inertia of a ring rotating in similar way. The moment of inertia of a solid sphere which has same radius as the disc and rotating in similar way, is n times higher than the moment of inertia of the given ring. Here, n =__________.

Consider all the bodies have equal masses.

Ans. (٤)

Sol.



$$I_1 = \frac{M_1R^2}{4}, I_2 = \frac{M_2R^2}{2}, I_3 = \frac{2M_1R^2}{5}$$

According to problem

$$\frac{I_1}{I_2} = 2.5\left(\frac{\frac{MR_1^2}{4}}{\frac{MR_2^2}{2}}\right) = \frac{5}{2}\left(\frac{R_1^2}{R_2^2}\right) = ٥ \ldots (١)$$

Now we are provided with information that

$$\frac{I_3}{I_2} = n$$

$$\Rightarrow \frac{\frac{2MR_1^2}{5}}{\frac{MR_2^2}{2}} = n \Rightarrow \frac{4R_1^2}{5R_2^2} = n \ \ldots\ldots (٢)$$

From Eq’، (١) and (٢) ⇒ n = ٤ In a measurement، it is asked to find modulus of

٤٨. elasticity per unit torque applied on the system.

The measured quantity has dimension of $M^a L^b T^c$.

If b = ٣، the value of c is_______

NTA Ans. (٤)

Sol. $\frac{\text{modulus of elasticity}}{\text{Torque}} = \frac{\text{Stress}}{\text{strain} \times \text{torque}}$

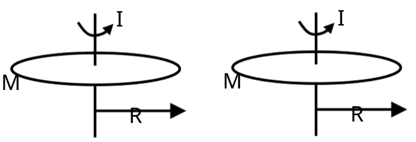
$= \frac{[\text{Force}]}{[\text{Area}][\text{Force} \times \text{length}]}$

$= \frac{1}{[\text{Area} \times \text{length}]} = [L^{-3}]$

٤٩. Two iron solid discs of negligible thickness have radii $R_١$ and $R_٢$ and moment of intertia $I_١$ and $I_٢$ respectively. For $R_٢ = ٢R_١$، the ratio of $I_١$ and $I_٢$ would be ١/x، where x = __________

Ans. (١٦)

Sol.



Given $R_٢ = ٢R_١$

$M_١ = (\pi R_١^٢)\, M_0$

$M_٢ = (\pi R_٢^٢)\, M_0$

$M_٢ = (\pi R_٢^٢) = \pi [2R_1]^2 = \pi 4R_1^2 = 4M_0$

$$\frac{I_1}{I_2} = \frac{\frac{M_1R_1^2}{2}}{\frac{M_2R_2^2}{2}} = \frac{M_1R_1^2}{M_2R_2^2} = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

٥٠. A double slit interference experiment performed with a light of wavelength ٦٠٠ nm forms an interference fringe pattern on a screen with ١٠th bright fringe having its centre at a distance of ١٠ mm from the central maximum. Distance of the centre of the same ١٠th bright fringe from the central maximum when the source of light is replaced by another source of wavelength ٦٦٠ nm would be______mm.

Ans. (١١)

Sol. In case of YDSE the distance of nth maxima from central maxima is given by

$Y = \frac{n\lambda D}{d}$

Here n، D & d are same

So، $y \propto \lambda$

$\Rightarrow \frac{y_2}{y_1} = \frac{\lambda_2}{\lambda_1} \Rightarrow \frac{y_2}{10\ \text{mm}} = \frac{660\ \text{nm}}{600\ \text{nm}}$

$\Rightarrow y_٢ = ١١$ mm

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON TUESDAY 28th JANUARY 2025) TIME : 9 : 00 AM TO 12 : 00 NOON

CHEMISTRY | TEST PAPER WITH SOLUTIONS

SECTION-A

٥١. The incorrect decreasing order of atomic radii is :

(١) Mg < Al < C < O (٢) Al < B < N < F

(٣) Be < Mg < Al < Si (٤) Si < P < Cl < F

Ans. (٣)

Sol. Correct order of atomic radii : Be > Mg < Al < Si

٥٢. Given below are two statements :

Statement I : In the oxalic acid vs $KMnO_4$ (in the presence of dil H_2SO_4) titration the solution needs to be heated initially to ٦٠°C, but no heating is required in Ferrous ammonium sulphate (FAS) vs $KMnO_4$ titration (in the presence of dil H_2SO_4)

Statement II : In oxalic acid vs $KMnO_4$ titration, the initial formation of $MnSO_4$ takes place at high temperature. Which then acts as catalyst for further reaction. In the case of FAS vs $KMnO_4$ heating oxidizes Fe^{2+} into Fe^{3+} by oxygen of air and error may be introduced in the experiment.

In the light of the above statements, choose the correct answer from the options given below :

(١) Statement I is false but Statement II is true

(٢) Both Statement I and Statement II are true

(٣) Statement I is true but Statement II is false

(٤) Both Statement I and Statement II are false

Ans. (٢)

Sol. For the titration : Oxalic acid v/s $KMnO_4$

$$2MnO_4^- + 5(COO)_2^{2-} + 16H^+ \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$$

This reaction is slow at room temperature, but becomes fast at ٦٠°C. Manganese(II) ions catalyse the reaction; thus, the reaction is autocatalytic; once manganese(II) ions are formed, it becomes faster and faster.

The titration of FAS v/s $KMnO_4$ do not require heating because at higher temperature the oxidation of Fe^{2+} to Fe^{3+} by atmospheric O_2 will be prominent.

٥٣. Match the List-I with List-II

	List-I (Redox Reaction)		List-II (Type of Redox Reaction)
A	$CH_4(g) + 2O_2(g) \rightarrow CO_2(g) + 2H_2O(l)$	(I)	Disproportionation reaction
B	$2NaH_{(s)} \rightarrow 2Na_{(s)} + H_{2(g)}$	(II)	Combination reaction
C	$V_2O_5(s) + 5Ca(s) \rightarrow 2V_{(s)} + 5CaO(s)$	(III)	Decomposition reaction
D	$2H_2O_2(aq) \rightarrow 2H_2O_{(l)} + O_2(g)$	(IV)	Displacement reaction

Choose the correct answer from the options given below :

(١) A-II, B-III, C-IV, D-I

(٢) A-II, B-III, C-I, D-IV

(٣) A-III, B-IV, C-I, D-II

(٤) A-IV, B-I, C-II, D-III

Ans. (١)

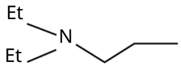
Sol. (A) Combustion of hydrocarbon

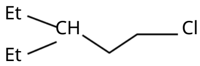
(B) Decomposition into gaseous product.

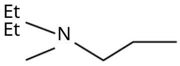
(C) Displacement of "V" by "Ca" atom.

(D) Disproportionation of H_2O_2 into O^{-2} and O^{0} oxidation states.

٥٤. Given below are two statements :

Statement I :  Clwill undergo alkaline hydrolysis at a faster rate than



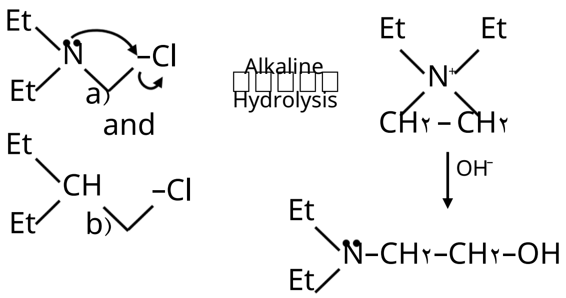
Statement II :  Cl، intramolecular substitution takes place first by involving lone pair of electrons on nitrogen.

In the light of the above statements، choose the most appropriate answer from the options given below :

(١)Both Statement I and Statement II are incorrect
(٢)Statement I is incorrect but statement II is correct
(٣)Both Statement I and Statement II are correct
(٤)Statement I is correct but Statement II is incorrect

Ans. (٣)

Sol.



Rate of(a) is faster than rate of(b)becauseit is a intramolecular substitution.

٥٥. A weak acid HA has degree of dissociation x. Which option gives the correct expression of pH – pKa)؟

(١)log (١ + ٢x)

(٢) $\log\left(\frac{1-x}{x}\right)$

(٣)٠

(٤) $\log\left(\frac{x}{1-x}\right)$

Ans. (٤)

Sol. HA ⇌ H^{+} + A^{-}

t=٠ a

t=t a(١–x) ax ax

Ka = (ax)$\frac{(x)}{1-x}$ ؛ [H^{+}] = ax

–log(Ka) = –log (ax) – log $\left(\frac{x}{1-x}\right)$

pKa = pH – $\log\left(\frac{x}{1-x}\right)$

pH – pKa = $\log\left(\frac{x}{1-x}\right)$

٥٦. Consider „n" is the number of lone pair of electrons present in the equatorial position of the most stable structure of ClF_3. The ions from the following with „n" number of unpaired electrons are :

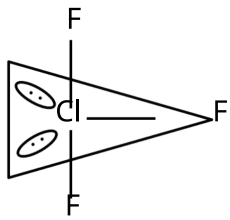
A. V^{3+} B. Ti^{3+}
C. Cu^{2+} D. Ni^{2+}
E. Ti^{2+}

Choose the correct answer from the options given below :

(١)A and C only (٢)A، D and E only
(٣)B and C only (٤)B and D only

Ans. (٢)

Sol. ClF_3



n = ٢ (No of lone pair presentinequitorial plane)

	(Unpaired e)
(A)V^{+3} : [Ar]$3d^2$	2
(B)Ti^{+3} : [Ar]$3d^1$	١
(C)Cu^{+2} : [Ar]$3d^9$	١
(D)Ni^{+2} : [Ar]$3d^8$	٢
(E)Ti^{+2} : [Ar]$3d^2$	٢

٥٧.

$[A]_0$ /mol L^{-1}	$t_{1/2}$ /min
٠.١٠٠	٢٠٠
٠.٠٢٥	١٠٠

For a given reaction R → P، $t_{1/2}$ is related to $[A]_0$ as given in table :

Given : log ٢ = ٠.٣٠

Which of the following is true ؟

A. The order of the reaction is $\frac{1}{2}$

B. If $[A]_0$ is ١M، then $t_{1/2}$ is ٢٠٠$\sqrt{10}$ min

C. The order of the reaction changes to ١ if the concentration of reactant changes from ٠.١٠٠ M to ٠.٥٠٠ M.

D. $t_{1/2}$ is ٨٠٠ min for $[A]_0$ = ١.٦ M

Choose the correct answer from the options given below :

(١)A and C only (٢)A and B only
(٣)A، B and D only (٤)C and D only

Ans.(٣)

Sol. $t_{1/2} \propto \frac{1}{A_0^{n-1}}$

$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \left(\frac{(A_0)_2}{(A_0)_1}\right)^{n-1}$

$\frac{200 \times 0.025}{100 \times 0.100} = \left(\frac{\;}{\;}\right)^{n-1}$

$2 = \left(\frac{1}{4}\right)^{n-1}$ $\qquad n-1 = -\frac{1}{2}$

$n = \frac{1}{2}$ (order)

$\therefore t_{1/2} \propto \sqrt{A_0}$

$\frac{200}{t_{1/2}} = \frac{(0.1)^{1/2}}{(1)^{1/2}}$ $\qquad$ when $A_0 = 1M$

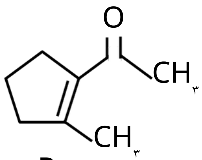
$t_{1/2} = 200\sqrt{10}$ min

❖ Ist order kinetics have $t_{1/2}$ independent of their concentration. So upon changing the concentration $t_{1/2}$ should not change for first order reaction.

$\frac{200}{t_{1/2}} = \frac{(0.1)^{1/2}}{(1.6)^{1/2}}$ $\qquad$ when $A_0 = 1.6M$

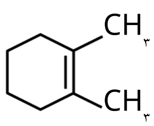
$t_{1/2} = 800$ min

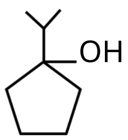
٥٨. A molecule ("P") on treatment with acid undergoes rearrangement and gives ("Q"). ("Q") on ozonolysis followed by reflux under alkaline condition gives ("R"). The structure of ("R") is given below :

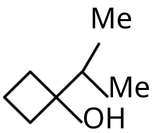


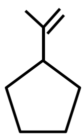
("R")

The structure of ("P") is

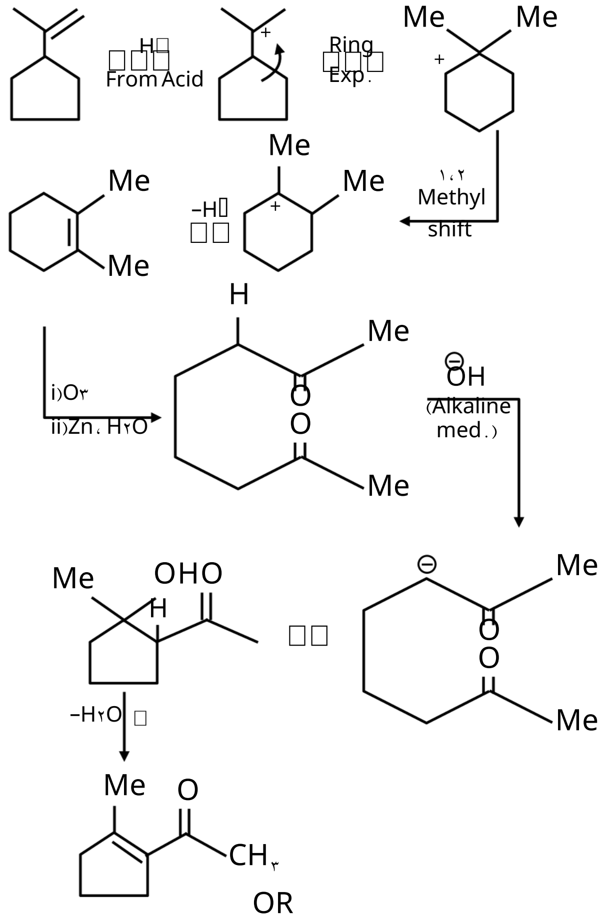
(1) 

(2) 

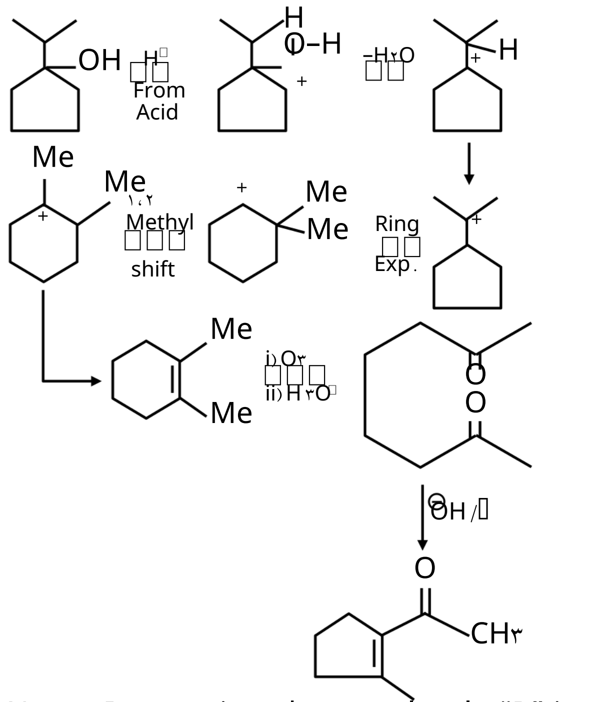
(3) 

(4) 

NTA Ans. (2)

Sol. 

OR

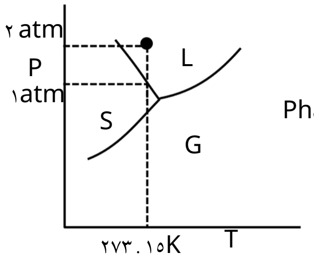


Note : In question about molecule "P" is not clarified, weather it is alcohol or alkene and as in question language rearrangement product is asking hence according to question language ans. is either (2) or (4). As alkene also undergoes rearrangement in presence of acid but option (2) also fulfil all conditions.

٥٩. Ice and water are placed in a closed container at a pressure of ١ atm and temperature ٢٧٣.١٥ K. If pressure of the system is increased ٢ times، keeping temperature constant، then identify correct observation from following : (١)Volume of system increases. (٢)Liquid phase disappears completely. (٣)The amount of ice decreases. (٤)The solid phase (ice) disappears completely. (٤)

Ans.

Sol.



Phase diagram of H_2O

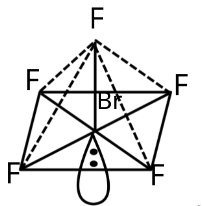
If pressure is made two time then mixture of ice and water will completely convert into water (liquid) form. The molecules having

٦٠. square pyramidal geometry are (١)BrF٥ & XeOF٤

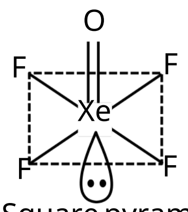
(٣)SbF٥ & PCl٥ (٢)SbF٥ & XeOF٤

(٤)BrF٥ & PCl٥

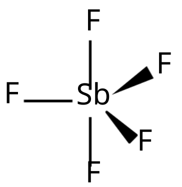
Ans.(١)



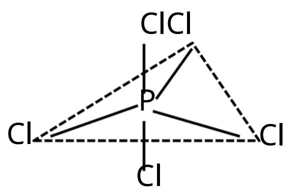
Square pyramidal



Square pyramidal



Trigonal Bipyramidal



Trigonal Bipyramidal

Sol.

BrF٥ : Square pyramedal

XeOF٤ : Square pyramedal

SbF٥ : Trigonal bipyramidal

PCl٥ : Trigonal bipyramidal

٦١. The metal ion whose electronic configuration is not affected by the nature of the ligand and which gives a violet colour in non-luminous flame under hot condition in borax bead test is(١)Ti ٣+ (٢) Ni٢+

(٣)Mn ٢+ (٤) Cr ٣+

Ans. (٢)

Sol. Ni gives violet coloured bead in non-luminous flame under hot conditions. Ni+٢ has ٨ configuration which does not depend on nature of ligand present in octahedral complex.

Ni+٢ : t٢g ٦ eg ٢

٦٢. Both acetaldehyde and acetone (individually) undergo which of the following reactions?

A. Iodoform Reaction

B. Cannizaro Reaction

C. Aldol condensation

D. Tollen's Test

E. Clemmensen Reduction

Choose the correct answer from the options given below :

(١)A، B and D only (٢)A، C and E only

(٣)C and E only (٤)B، C and D only

Ans. (٢)

Sol.

S.No.	Name of Reaction	Acetaldehyde $CH_3-\overset{O}{\overset{\|\|}{C}}-H$	Acetone $CH_3-\overset{O}{\overset{\|\|}{C}}-CH_3$
١	Iodoform reaction	ve	ve
٢	Cannizaro	⊖ve	⊖ve
٣	Aldol reaction	ve	ve
٤	Tollen"s test	ve	⊖ve
٥	Clemmensen reduction	ve	ve

Ans. (٢) A، C and E only

٦٣. In a multielectron atom، which of the following orbitals described by three quantum numbers with have same energy in absence of electric and magnetic fields؟

A. n = ١، ١ = ٠، m١ = ٠

B. n = ٢، ١ = ٠، m١ = ٠

C. n = ٢، ١ = ١، m١ = ١

D. n = ٣، ١ = ٢، m١ = ١

E. n = ٣، ١ = ٢، m١ = ٠

Choose the correct answer from the options given below :

(١) A and B only

(٢) B and C only

(٣) C and D only

(٤) D and E only

Ans. (٤)

Sol.

	orbital
A : n = ١، ℓ = ٠، mℓ = ٠	١s
B : n = ٢، ℓ = ٠، mℓ = ٠	٢s
C : n = ٣، ℓ = ١، mℓ = ١	٣p
D : n = ٣، ℓ = ٢، mℓ = ١	٣d
E : n = ٣، ℓ = ٢، mℓ = ٠	٣d

In absence of electric and magnetic fields، all orbitals of ٣d are degenerate

٦٤. The products A and B in the following reactions، respectively are

A ⟵(AgNO٢) CH3 CH2 CH2 Br ⟶(AgCN) B

(١) CH٣–CH٢–CH٢–ONO، CH٣–CH٢–CH٢–NC

(٢) CH٣–CH٢–CH٢–ONO، CH٣–CH٢–CH٢–CN

(٣) CH٣–CH٢–CH٢–NO٢، CH٣–CH٢–CH٢–CN

(٤) CH٣–CH٢–CH٢–NO٢، CH٣–CH٢–CH٢–NC

Ans. (٤)

Sol. CH٣–CH٢–CH٢–NO٢ ⟵(AgNO٢) CH٣–CH٢–CH٢–Br
(A)

⟶(AgCN) CH٣–CH٢–CH٢–NC
(B)

٦٥. What is the freezing point depression constant of a solvent، ٥٠ g of which contain ١ g non volatile solute (molar mass ٢٥٦ g mol⁻¹) and the decrease in freezing point is ٠.٤٠ K؟

(١) ٥.١٢ K kg mol⁻¹ (٢) ٤.٤٣ K kg mol⁻¹

(٣) ١.٨٦ K kg mol⁻¹ (٤) ٣.٧٢ K kg mol⁻¹

Ans. (١)

Sol. ΔTf = Kb.m

$$0.4 = K_b \frac{\frac{1}{256}}{50 \times 10^{-3}}$$

Kb = ٥.١٢ K kg / mol

٦٦. Consider the following elements In، Tl، Al، Pb، Sn and Ge.
The most stable oxidation states of elements with highest and lowest first ionisation enthalpies، respectively، are

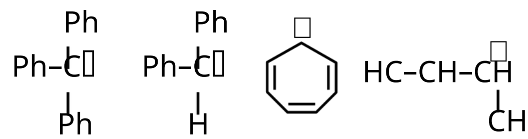
(١) +٢ and +٣ (٢) +٤ and +٣

(٣) +٤ and +١ (٤) +١ and +٤

NTA Ans. (٣)

Sol. Among Al، In، Tl، Ge، Sn، Pb، the metal having highest IE١ is Ge and lowest IE١ is In.
Most stable oxidation state of Ge is +٤ and In is +٣.

٦٧. The correct order of stability of following carbocation is :



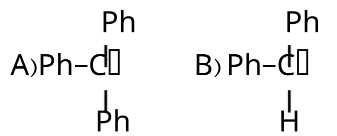
A B C D

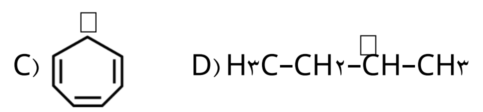
(١) A < B < C < D (٢) B < C < A < D

(٣) C < B < A < D (٤) (٤) C < A < B < D

Ans.

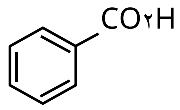
Sol.

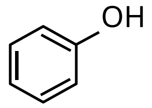


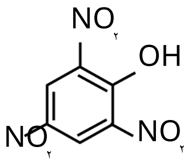


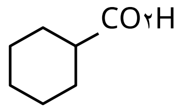
Solution :-
C is aromatic due to ⊖ve charge hence it is most stable
A have more resonance structure
B have less resonance structure
D have only hyper conjugation
Consider First Aromaticity < Resonance < Hyper conjugation
Ans. D > B > A > C

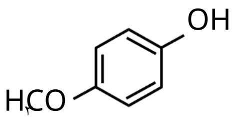
٦٨. The compounds that produce $CO_٢$ with aqueous $NaHCO_٣$ solution are :

A. 

B. 

C. 

D. 

E. 

Choosethe correctanswer from the options given below :

(١)A and C only (٢)A، B and E only

(٣)A، C and D only (٤)A and B only

Ans. (٣)

Sol. A، C، D produce $CO_٢$ with aqueous $NaHCO_٣$ solution.

A، C، D acids are stronger acid than $H_٢CO_٣$ (Carbonic acid)

٦٩. Which of the following oxidation reactions are carried out by both $K_٢Cr_٢O_٧$ and $KMnO_٤$ in acidic medium ?

A. I → I٢

B. S → S

C. Fe → Fe ٣+

D. I → IO٣ −

E. S٢O٣ → SO٤ ٢−

Choose the correct answer from the options given below :

(١)B، C and D only (٢)A، D and E only

(٣)A، B and C only (٤)C، D and E only

Ans. (٣)

Sol. I → H → I٢ I → OH → IO٣

S2 → H → S SO٢٣ → OH → SO٢٤

Fe٢ → Fe٣

SO٢٣ → H → S SO٢٤

٧٠. Given below are two statements :

Statement I : D-glucose pentaacetate reacts with ٢، ٤-dinitrophenylhydrazine.

Statement II : Starch، on heating with concentrated sulfuric acid at ١٠٠°C and ٢-٣ atmosphere pressure produces glucose.

In the light of the above statements، choose the correct answer from the options given below

(١)Both Statement I and Statement II are false

(٢)Statement I is false but Statement II is true

(٣)Statement I is true but Statement II is false

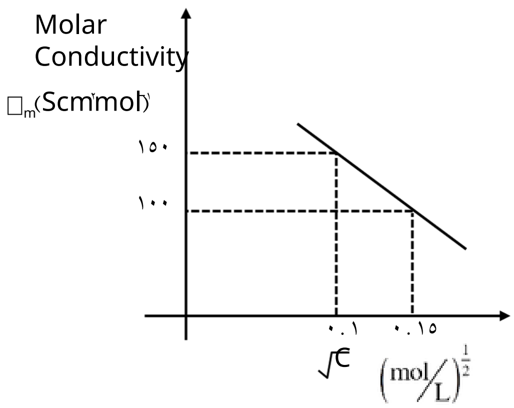
(٤)Both Statement I and Statement II are true

Ans. (٢)

Sol.

SECTION-B

٧١. Given below is the plot of the molar conductivity vs $\sqrt{\text{concentration}}$ for KCl in aqueous solution.



If، for the higher concentration of KCl solution، the resistance of the conductivity cell is ١٠٠Ω، then the resistance of the same cell with the dilute solution is "x"Ω. The value of x is ____________ (Nearest integer)

Ans. ١٥٠

Sol. $R = \rho \frac{\ell}{A}$

$\kappa = G.G^*$ $\quad G = \frac{1}{R}; \kappa = \frac{1}{\rho}$

$G^* = \frac{\ell}{A}$

R = Resistance

ρ = Resistivity

$\frac{\ell}{A}$ = cell constant (G^*)

$\frac{\kappa_c}{\kappa_d} = \frac{R_d}{R_c}; \Lambda_m = \frac{\kappa \times 1000}{C}$

$\frac{\kappa_c}{\kappa_d} = \frac{(\Lambda_m.C)_c}{(\Lambda_m.C)_d} = \frac{R_d}{R_c}$ $\quad$ c = concentrated sol. d = diluted solution

$\frac{100.(0.15)^2}{150.(0.1)^2} = \frac{R_d}{100}$

$R_d = 150\ \Omega$

72. Quantitative analysis of an organic compound (X) shows following % composition.

C : 14.5% $\qquad$ Cl : 64.46%

H : 1.8%

(Empirical formula mass of the compound (X) is ________ $\times 10^{-1}$

(Given molar mass in g mol^{-1} of C : 12, H : 1, O : 16, Cl : 35.5)

Ans. 1655

Sol.

	C	Cl	H	O
%mass	14.5	64.46	1.8	19.24
Molar ratio	$\frac{14.5}{12}$	$\frac{64.46}{35.5}$	$\frac{1.8}{1}$	$\frac{19.24}{16}$
	1.2	1.8	1.8	1.2
Minimum integral ratio	2	3	3	2

Empiricial formula = $C_2H_3Cl_3O_2$

Mass = 165.5

Mass = 1655×10^{-1}

73. The molarity of a 70% (mass/mass) aqueous solution of a monobasic acid (X) is __________ M (Nearest integer)

[Given : Density of aqueous solution of (X) is 1.25 g mL^{-1}

Molar mass of the acid is 70 g mol^{-1}]

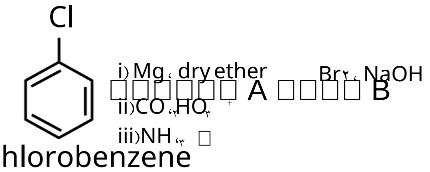
Ans. 125

Sol. Assuming 100 gm solution contain 70 gm solute.

Volume of 100 gm solution will be $\frac{100}{1.25}$ ml.

Molarity = $\frac{70/70}{100/1.25} \times 1000 = 12.5$ or 125×10^{-1}

74. Consider the following sequence of reactions :



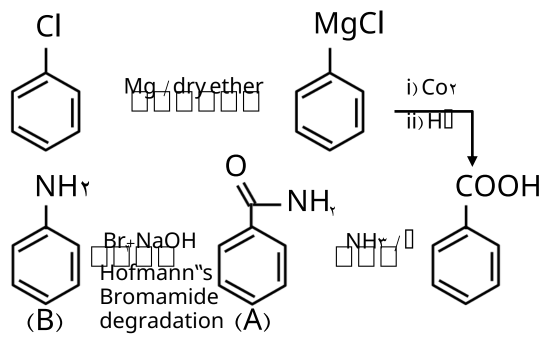
Chlorobenzene

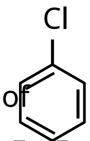
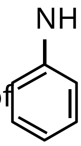
11.25 mg of chlorobenzene will produce ________ $\times 10^{-1}$ mg of product B. (Consider the reactions result in complete conversion.)

[Given molar mass of C, H, O, N and Cl as 12, 1, 16, 14 and 35.5 g mol^{-1} respectively]

Ans. 93

Sol.



No. of moles of  = No. of moles of 

$\frac{11.25 \times 10^{-3}}{112.5} = \frac{x \times 10^{-1} \times 10^{-3}}{93}$

$x \times 10^{-1} = 93 \times 0.1$

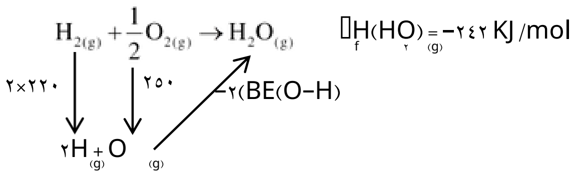
x = 93 mg

75. The formation enthalpies, ΔH_f° for H(g) and O(g) are 220.0 and 250.0 kJ mol^{-1}, respectively, at 298.15 K, and ΔH_f° for H_2O(g) is –242.0 kJ mol^{-1} at the same temperature. The average bond enthalpy of the O–H bond in water at 298.15 K is __________ kJ mol^{-1} (nearest integer).

Ans. 466

Sol. $\frac{1}{2}H_{2(g)} \rightarrow H(g)$ $\quad$; $\Delta H_f(H) = 220$ KJ/mol

$\frac{1}{2}O_{2(g)} \rightarrow O(g)$ $\quad$; $\Delta H_f(O) = 250$ KJ/mol



$\Delta H_f(H_2O(l)) = -242 = 440 + 250 - 2(B.E.(O–H))$

BE(O–H) = 466 KJ/mol