5. Let

$A = \left\{x \in (0, \pi) - \left\{\frac{\pi}{2}\right\} : \log_{(2/\pi)}|\sin x| + \log_{(2/\pi)}|\cos x| = 2\right\}$

and

$B = \{x \ge 0 : \sqrt{x}(\sqrt{x} - 4) - 3|\sqrt{x} - 2| + 6 = 0\}$. Then

$n(A \cup B)$ is equal to :

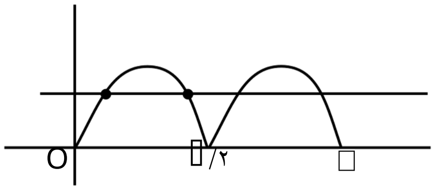
(1) 4 (3) 8 (2) 2 (4) 6

Ans. (3)

Sol. A : $\log_{2/\pi}|\sin x| + \log_{2/\pi}|\cos x| = 2$

$\Rightarrow \log_{2/\pi}(|\sin x \cdot \cos x|) = 2$

$\Rightarrow |\sin 2x| = \frac{8}{\pi^2}$



Number of solution 4

B : let $\sqrt{x} = t > 2$

Then $\sqrt{x}(\sqrt{x} - 4) - 3(\sqrt{x} - 2) + 6 = 0$

$\Rightarrow t^2 - 4t + 3t - 6 + 6 = 0$

$\Rightarrow t^2 - t = 0$, $t = 0$, $t = 1$

$x = 0$, $x = 1$

again let $\sqrt{x} = t \ge 2$

then $t^2 - 4t - 3t + 6 + 6 = 0$

$\Rightarrow t^2 - 7t + 12 = 0$

$\Rightarrow t = 3, 4$

$x = 9, 16$

Total number of solutions

$n(A \cup B) = 4 + 4 = 8$

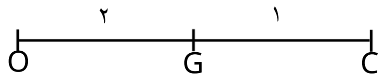
6. Let the position vectors of three vertices of a triangle be $4\vec{p} + \vec{q} - 3\vec{r}$, $-5\vec{p} + \vec{q} + 2\vec{r}$ and $2\vec{p} - \vec{q} + 2\vec{r}$. If the position vectors of the orthocenter and the circumcenter of the triangle are $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$ and $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$ respectively, then $(\alpha + 2\beta + 5\gamma)$ is equal to :

(1) 3 (2) 1

(3) 6 (4) 4

Ans. (1)

Sol. We know that



O (orthocentre) $\frac{\vec{p} + \vec{q} + \vec{r}}{4}$

C (circum centre) $\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}$

C (centroid) = $\frac{\vec{p} + \vec{q} + \vec{r}}{3}$

by relation

$2(\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}) + \frac{\vec{p} + \vec{q} + \vec{r}}{4} = 3\left(\frac{\vec{p} + \vec{q} + \vec{r}}{3}\right)$

$8(\alpha\vec{p} + \beta\vec{q} + \gamma\vec{r}) = 3(\vec{p} + \vec{q} + \vec{r})$

$\Rightarrow 8\alpha = 3$, $8\beta = 3$, $8\gamma = 3$

$\alpha = \frac{3}{8}$, $\beta = \frac{3}{8}$, $\gamma = \frac{3}{8}$

$\therefore \alpha + 2\beta + 5\gamma$

$\frac{3}{8} + \frac{6}{8} + \frac{15}{8} = \frac{24}{8} = 3$

7. Let $[x]$ denote the gereatest integer function and let m and n respectively be the numbers of the points, where the function $f(x) = [x] + |x - 2|$, $-2 < x < 3$, is not continuous and not differentiable.

Then $m + n$ is equal to :

(1) 6 (3) 8

(2) 9 (4) 7

Ans. (3)

Sol. $f(x) = [x] + |x - 2|$ $-2 < x < 3$

$f(x) = \begin{cases} -x, & -2 < x < -1 \\ -x + 1, & -1 \le x < 0 \\ 2 - x, & 0 \le x < 1 \\ \ldots, & 1 \le x < 2 \\ x - 3, & 2 \le x < 3 \\ x, & \ldots \end{cases}$

So f(x) is not continuous at 4 points and not differentiable at 4 point

So $m + n = 4 + 4 = 8$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON FRIDAY 24th JANUARY 2025) TIME : 3:00 PM TO 6:00 PM

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The equation of the chord, of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid-point is (3, 1) is :

(1) $48x + 25y = 169$ (2) $4x + 122y = 134$

(3) $25x + 101y = 176$ (4) $5x + 16y = 31$

Ans. (1)

Sol. Equation of chord with given middle point

$T = S_1$

$\Rightarrow \frac{3x}{25}+\frac{y}{16}-1=\frac{9}{25}+\frac{1}{16}-1$

$48x + 25y = 144 + 25$

$48x + 25y = 169$ Ans.

2. The function $f : (-\infty, \infty) \to (-1, 1)$ defined by $f(x)=\frac{2^x-2^{-x}}{2^x+2^{-x}}$ is :

(1) One-one but not onto

(2) Onto but not one-one

(3) Both one-one and onto

(4) Neither one-one nor onto

Ans. (1)

Sol. $f(x)=\frac{2^{2x}-1}{2^{2x}+1}$

$=1-\frac{2}{2^{2x}+1}$

$f'(x)=\frac{2}{(2^{2x}+1)^2}\cdot 2\cdot 2^{2x}\cdot \ln 2$ i.e. always +ve

so $f(x)$ is $\uparrow$ function

$\therefore f(-\infty) = -1$

$f(\infty) = 1$

$\therefore f(x) \in (-1, 1) \ne$ co-domain

so function is one-one but not onto

3. If $\alpha > \beta > \gamma > 0$, then the expression

$\cot^{-1}\left(\beta+\frac{(1+\beta^2)}{(\alpha-\beta)}\right)+\cot^{-1}\left(\gamma+\frac{(1+\gamma^2)}{(\beta-\gamma)}\right)+\cot^{-1}\left(\alpha+\frac{(1+\alpha^2)}{(\gamma-\alpha)}\right)$ is equal to :

(1) $\frac{\pi}{2}-(\alpha+\beta+\gamma)$ (2) 3π

(3) 0 (4) π

Ans. (4)

Sol. $\because \cot^{-1}\left(\frac{1+\alpha\beta}{\alpha-\beta}\right)+\cot^{-1}\left(\frac{1+\beta\gamma}{\beta-\gamma}\right)+\cot^{-1}\left(\frac{1+\gamma\alpha}{\gamma-\alpha}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\alpha-\beta}{1+\alpha\beta}\right)+\tan^{-1}\left(\frac{\beta-\gamma}{1+\beta\gamma}\right)+\pi+\tan^{-1}\left(\frac{\gamma-\alpha}{1+\gamma\alpha}\right)$

$\Rightarrow \tan^{-1}\alpha-\tan^{-1}\beta+\tan^{-1}\beta-\tan^{-1}\gamma+\pi+\tan^{-1}\gamma-\tan^{-1}\alpha$

$= \pi$

4. Let $f : (0, \infty) \to R$ be a function which is differentiable at all points of its domain and satisfies the condition $x^2f'(x) = 2xf(x) + 3$, with $f(1) = 4$. Then $2f(2)$ is equal to :

(1) 29 (2) 19

(3) 39

(4) 23

Ans. (3)

Sol. $x^2f'(x) - 2x f(x) = 3$

$\Rightarrow \frac{x^2f'(x)-2xf(x)}{(x^2)^2}=\frac{3}{x^4}$

$\Rightarrow \frac{d}{dx}\left(\frac{f(x)}{x^2}\right)=\frac{3}{x^4}$

Integrating both sides

$\frac{f(x)}{x^2}=-\frac{1}{x^3}+C$

$f(x) = -\frac{1}{x}+Cx^2$

put $x = 1$

$4 = -1 + C \Rightarrow C = 5$

$f(x) = -\frac{1}{x}+5x^2$

Now $2 \times f(2) = 2\left(-\frac{1}{2}+5\cdot 2^2\right)$

$= 39$

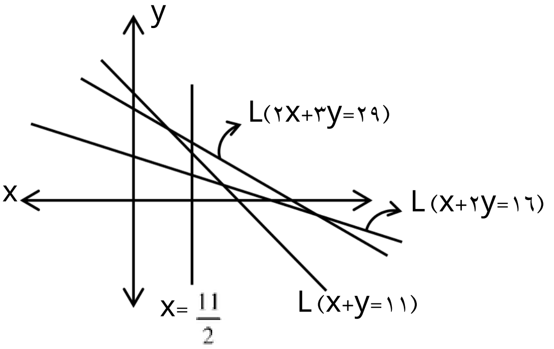
٨. Let the points $\left(\frac{11}{2}, \alpha\right)$ lie on or inside the triangle with sides x + y = ١١، x + ٢y = ١٦ and ٢x + ٣y = ٢٩. Then the product of the smallest and the largest values of □ is equal to :

(١) ٢٢ (٢) ٤٤

(٣) ٣٣ (٤) ٥٥

Ans.(٣)

Sol.



Point of intersection of $x = \frac{١١}{٢}$ with L & L gives، $\alpha_{min} = \frac{١١}{٢}$

and α_{max} = ٦

$\therefore \alpha_{min} \cdot \alpha_{max} = \frac{١١}{٢} \times ٦ = ٣٣$

٩. In an arithmetic progression، if $S_{٤٠}$ = ١٠٣٠ and $S_{١٢}$ = ٥٧، then $S_{٣٠} - S_{١٠}$ is equal to :

(١) ٥١٠ (٢) ٥١٥

(٣) ٥٢٥ (٤) ٥٠٥

Ans. (٢)

Sol. Let a & d are first term and common diff of an AP.

$S_{٤٠} = \frac{٤٠}{٢}[٢a + ٣٩d] = ١٠٣٠$ ... (١)

$S_{١٢} = \frac{١٢}{٢}[٢a + ١١d] = ٥٧$ ... (٢)

by (١) & (٢)

a = □ $\frac{٧}{٢}$ d = $\frac{٣}{٢}$

$\therefore S_{٣٠} - S_{١٠} = \frac{٣٠}{٢}[2a + 29d] - \frac{١٠}{٢}[٢a + ٩d]$

= ٢٠a + ٣٩٠d

= ٥١٥

١٠. If $٧ = ٥ + \frac{1}{7}(٥ + \alpha) + \frac{1}{7^2}(٥ + ٢\alpha) + \frac{1}{7^3}(٥ + ٣\alpha) + \ldots\ldots$ ∞، then the value of □ is :

(١) ١ (٢) $\frac{6}{7}$

(٣) ٦ (٤) $\frac{1}{7}$

Ans. (٣)

Sol. Let $S = ٥ + \frac{١}{٧}(٥ + \alpha) + \frac{١}{٧^٢}(٥ + ٢\alpha) + \ldots$

$\frac{١}{٧}S = \frac{١}{٧}(5) + \frac{١}{٧^2}(٥ + \alpha) + \ldots$

$\frac{٦}{٧}S = ٥ + \alpha\left(\frac{\frac{١}{٧}}{١ - \frac{١}{٧}}\right)$

6 = 5 + $\frac{\alpha}{٦}$ ⇒ α = 6

١١. If the system of equations x + ٢y − ٣z = ٢، ٢x + λy + ٥z = ٥، ١٤x + ٣y + μz = ٣٣ has infinitely many solutions، then λ + μ is equal to :

(١) ١٣ (٢) ١٠

(٣) ١١ (٤) ١٢

Ans.

Sol. $D = \begin{vmatrix} ١ & ٢ & -3 \\ ٢ & \lambda & ٥ \\ ١٤ & ٣ & \mu \end{vmatrix} = ٠$، $\lambda\mu + ٤٢\lambda - ٤\mu + ١٠٧ = ٠$

$D_١ = ٢\lambda\mu + ٩٩\lambda - ١٠\mu + ٢٥٥$

$D_٢ = ١٣ - \mu$

$D_٣ = ٥\lambda + ٥$

$D_٢ = ٠ \Rightarrow \mu = ١٣$ & $D_٣ = ٠ \Rightarrow \lambda = -١$

check & verify for these values D & $D_١ = ٠$

١٢. Let (٢، ٣) be the largest open interval in which the function $f(x) = ٢\log_e(x - ٢) - x^٢ + ax + ١$ is strictly increasing and (b، c) be the largest open interval، in which the function $g(x) = (x - ١)^٣(x + ٢ - a)^٢$ is strictly decreasing. Then ١٠٠(a + b − c) is equal to :

(١) ٢٨٠ (٢) ٣٦٠

(٣) ٤٢٠ (٤) ١٦٠

Ans.(٢)

Sol. $f'(x) = \frac{2}{x-2} - 2x + a \geq 0$

$f''(x) = \frac{-2}{(x-2)^2} - 2 \leq 0$

$f'(x)$

$f'(3) \geq 0$

$2 - 6 + a \geq 0$

$a \geq 4$

$a_{min} = 4$

$g(x) = (x-1)(x+2-a)^2$

$g(x) = (x-1)(x-2)^2$

$g'(x) = (x-1)\,2(x-2) + (x-2)^2(x-1)$

$= (x-1)(x-2)(2x-2+2x-6)$

$= (x-1)(x-2)(5x-8) > 0$

$x \in \left(\frac{8}{5}, 2\right)$

$100(a+b-c) = 100\left(4 + \frac{8}{5} - 2\right) = 360$

13. Suppose A and B are the coefficients of 30^{th} and 12^{th} terms respectively in the binomial expansion of $(1+x)^{2n-1}$. If $2A = 5B$, then n is equal to:

(1) 22 (2) 21

(3) 20 (4) 19

Ans. (2)

Sol. $A = {}^{2n-1}C_{29}$ $B = {}^{2n-1}C_{11}$

$2\,{}^{2n-1}C_{29} = 5\,{}^{2n-1}C_{11}$

$2\frac{(2n-1)!}{29!(2n-30)!} = 5\frac{(2n-1)!}{(2n-12)!\,11!}$

$\frac{2}{29\ldots12} = 5\frac{1}{(2n-12)(2n-13)\ldots(2n-29)}$

$\frac{1}{30\cdot29\ldots12} = \frac{1}{(2n-12)(2n-13)\ldots(2n-29)}$

$2n - 12 = 30$

$n = 21$

14. Let $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$ and $\vec{c} = \vec{b} \times \hat{k}$. Then the projection of $\vec{c} - 2\hat{j}$ on $\vec{a}$ is:

(1) $3\sqrt{7}$

(2) $\sqrt{14}$

(3) $2\sqrt{14}$

(4) $2\sqrt{7}$

Ans. (3)

Sol. $\vec{b} = \vec{a} \times (\hat{i} - 2\hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = 2\hat{i} + 8\hat{j} + \hat{k}$

$\vec{c} = \vec{b} \times \hat{k} = 8\hat{i} \ldots 2\hat{j}$

$\vec{c} = -2\hat{j} + 8\hat{i} - 4\hat{j}$ (?)

Projection of $(\vec{c} - 2\hat{j})$ on $\vec{a}$

$\frac{(\vec{c} - 2\hat{j}) \cdot \vec{a}}{|\vec{a}|} = \frac{\langle 8, -4, 0\rangle \cdot \langle 3, -1, 2\rangle}{\sqrt{14}}$

$= \frac{28}{\sqrt{14}} = 2\sqrt{14}$

15. For some a, b, let

$$f(x) = \begin{vmatrix} a + \frac{\sin x}{x} & 1 & b \\ a & 1 + \frac{\sin x}{x} & b \\ a & 1 & b + \frac{\sin x}{x} \end{vmatrix}, \quad x \neq 0,$$

$\lim_{x\to0} f(x) = \lambda + \mu a + \nu b$. Then $(\lambda + \mu + \nu)^2$ is equal to:

(1) 25 (2) 9

(3) 36 (4) 16

Ans. (4)

Sol. $\lim_{x\to0} f(x) = \begin{vmatrix} a+1 & 1 & b \\ a & 1+1 & b \\ a & 1 & b+1 \end{vmatrix}$

$= (a+1)(2(b+1) - b) + 1(ab - a(b+1)) + ba$

$= (a+1)(b+2) - a + ab$

$= b + a + 2 = \lambda + \mu a + \nu b$

$\lambda = 2, \mu = 1, \nu = 1 \Rightarrow (\lambda + \mu + \nu)^2 = 16$

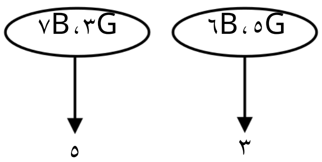
16. Group A consists of 7 boys and 3 girls, while group B consists of 6 boys and 5 girls. The number of ways, 4 boys and 4 girls can be invited for a picnic if 5 of them must be from group A and the remaining 3 from group B, is equal to:

(1) 8575 (2) 9100

(3) 8925 (4) 8750

Ans. (٣)

Sol.



C–I (٣G &٢B)& (١G & ٢B) (٢G &

C–II ٢B)& (٢G & ١B) (١G & ٤B)&

C–III (٣G & ٠B)

Total = C–I + C–II + C–III

= C ٢ . C ٢ . C ٦ . C ٤ + C ٢ . C ٢ . C C + C ٥ . C ٢ . C ٤ . C

= ٨٩٢٥

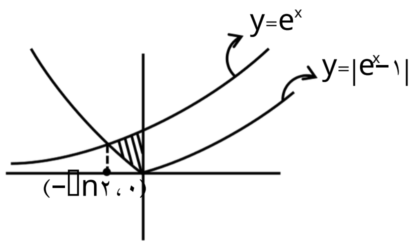
١٧. The area of the region enclosed by the curves $y = e^x$, $y = |e^x - 1|$ and y-axis is:

(١) ١ + log٢ e (٢) log٢ e

(٣) ٢ log٢ e – ١ (٤) ١ – log٢ e

Ans. (٤)

Sol.



For Area $\int_{-\ell n 2}^{0} (e^x - (1 - e^x))\,dx$

$\int_{-\ell n 2}^{0} (2e^x - 1)\,dx = (2e^x - x)\Big|_{-\ell n 2}^{0}$

= (٢ – (١ + ℓn٢))

= ١ – ℓn٢

١٨. The number of real solution(s) of the equation $x^2 + 3x + 2 = \min\{|x - 3|, |x + 2|\}$ is:

(١) ٢

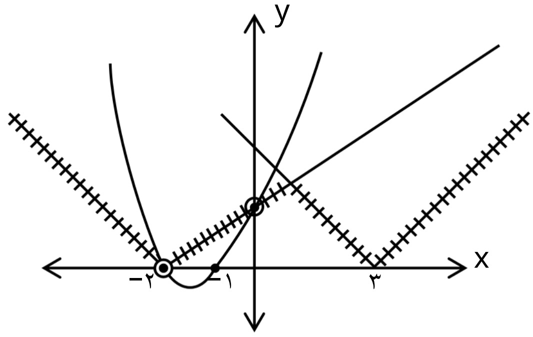
(٢) ٠

(٣) ٣

(٤) ١

Ans. (١)

Sol.



Only ٢ solutions.

١٩. Let A = [a_ij] be a square matrix of order ٢ with entries either ٠ or ١. Let E be the event that A is an invertible matrix. Then the probability P(E) is:

(١) $\frac{5}{8}$ (٢) $\frac{3}{16}$

(٣) $\frac{1}{8}$ (٤) $\frac{3}{8}$

Ans. (٤)

Sol. C–I $\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$ → ٤ ways

C–II $\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$ & $\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$ → ٢ ways

P = favourable / total = ٦/١٦ = ٣/٨

٢٠. If the equation of the parabola with vertex $V\left(\frac{3}{2}, 3\right)$ and the directrix x + ٢y = ٠ is $\alpha x^2 + \beta y^2 - \gamma xy - 30x - 60y + 225 = 0$, then α + β + γ is equal to:

(١) ٦

(٢) ٨

(٣) ٧

(٤) ٩

Ans. (٤)

Sol. Equation of axis $y - 3 = 2\left(x - \frac{3}{2}\right)$

y – ٢x = ٠

foot of directrix

y – ٢x = ٠

& → (٠، ٠)

٢y + x = ٠

Focus = (٣، ٦)

$PS^2 = PM^2$

$$(x-3)^2 + (y-6)^2 = \left(\frac{x + 2y}{\sqrt{5}}\right)^2$$

$4x^2 + y^2 - 4xy - 30x - 60y + 225 = 0$

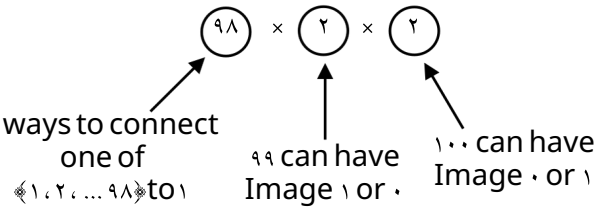
$\alpha = 4, \beta = 1, \gamma = 4$ $\alpha + \beta + \gamma = 9$

SECTION-B

٢١. Number of functions $f : \{1, 2, \ldots, 100\} \to \{0, 1\}$, that assign ١ to exactly one of the positive integers less than or equal to ٩٨، is equal to _____.

Ans. (٣٩٢)

Sol.

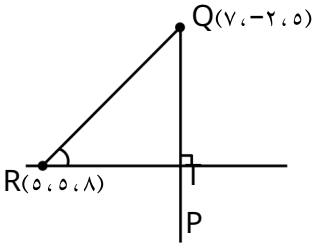


٣٩٢ Ans.

٢٢. Let P be the image of the point Q(٧، −٢، ٥) in the line $L: \frac{x-1}{2} = \frac{y-1}{3} = \frac{z}{4}$ and R(٥، p، q) be a point on L. Then the square of the area of ΔPQR is _____

Ans. (٩٥٧)

Sol.



Let R(٢α + ١، ٣α − ١، ٤α)

٢α + ١ = ٥

α = ٢

R(٥، ٥، ٨)

let T(٢α + ١، ٣α − ١، ٤α)

$\overrightarrow{QT} = (2\alpha - 6)\hat{i} + (3\alpha - 1)\hat{j} + (4\alpha - 5)\hat{k}$

$\vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{QT} \cdot \vec{b} = 0$

$4(-12 + 9\alpha + 3 + 16\alpha - 20) = 0$

α = ١

T(٣، ٢، ٤)

$QT = \sqrt{33}$ $RT = \sqrt{29}$

$(\text{area of } \Delta PQR) = \frac{1}{2}\sqrt{29\sqrt{33}^2}$

= ٩٥٧

٢٣. Let y = y(x) be the solution of the differential equation $2\cos x \frac{dy}{dx} = \sin 2x - 4y\sin x$، $x \in \left(0, \frac{\pi}{2}\right)$.

If $y\left(\frac{\pi}{3}\right) = 0$، then $y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right)$ is equal to _____.

Ans. (١)

Sol. $\frac{dy}{dx} = 2y\tan x - \sin x$

$I.F. = e^{\int -2\tan x dx} = \sec^2 x$

$y\sec^2 x = \int \sin x \sec^2 x \, dx$

$= \int \tan x \sec x \, dx$

$= \sec x + C$

$C = -2$

$y = \cos x - 2\cos^2 x$

$y\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - 1$

$y' = -\sin x + 4\cos x \sin x$

$y'\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}} - 2$

$y'\left(\frac{\pi}{4}\right) + y\left(\frac{\pi}{4}\right) = 1$

٢٤. Let $H_1: \frac{x^2}{\frac{A^2}{2}} - \frac{y^2}{\frac{B^2}{2}} = 1$ and $H_2: \frac{x^2}{A^2} - \frac{y^2}{B^2} = 1$ be two hyperbolas having length of latus rectums $\sqrt{108}$ and $12\sqrt{5}$ respectively. Let their eccentricities be e_1 and $\sqrt{\frac{5}{2}}$ and e_2 respectively. If the product of the lengths of their transverse axes is $10\sqrt{10}$، then $25e_2^2$ is equal to _____

Ans. (٥٥)

Sol. $\frac{b^2}{a^2} = 10\sqrt{2}$

$1 - \frac{b^2}{a^2} = \frac{5}{2}$

$a = 5\sqrt{2}$

$b = 5\sqrt{3}$

$\frac{2A^2}{B} = 12\sqrt{5}$

$2a.2B = 100\sqrt{10}$

$2.5\sqrt{2}.2B = 100\sqrt{10}$

$B = 5\sqrt{5}$

$A = 5\sqrt{6}$

$e_2^2 = 1 - \frac{A^2}{B^2}$

$= 1 - \frac{150}{125}$

$e_2^2 = 1 - \frac{30}{25}$

$25e_2^2 = 55$

25. If $\int \frac{2x^2 - 5x + 9}{\sqrt{x^2 - x + 1}}dx = x\sqrt{x^2 - x + 1} + \sqrt{x^2 - x + 1} - 3\log_e\left|x - \frac{1}{2} + \sqrt{x^2 - x + 1}\right| + C$, where C is the constant of integration, then $\alpha + 2$ is equal to ____

Ans. (16)

Sol. $2x^2 + 5x + 9 = A(x + x^2 + 1) + B(2x + 1) + C$

$A = 2 \qquad B = \frac{3}{2} \qquad C = \frac{11}{2}$

$2\int\sqrt{x^2 - x + 1}dx = 2\int\frac{2x - 1}{\sqrt{x^2 - x + 1}}dx + 11\int\frac{dx}{\sqrt{x^2 - x + 1}}$

$2\int\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}dx = 2\sqrt{x^2 - x + 1} + 11\int\frac{dx}{\sqrt{\left(x - \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}}$

$2\left(\frac{x - \frac{1}{2}}{2}\right)\sqrt{x^2 - x + 1} + 3\ell n\left|x - \frac{1}{2}\right| + \sqrt{x^2 - x + 1}\left|+ 11\ell n\right|x - \frac{1}{2} + \sqrt{x^2 - x + 1}\left| + C\right.$

$= \frac{11}{2}\ell n\left|x - \frac{1}{2} + \sqrt{x^2 - x + 1}\right| + C$

$\alpha = \frac{7}{2} \qquad \beta = \frac{25}{4}$

$\alpha + 2\beta = 16$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON FRIDAY24 th JANUARY2025) TIME : 3 : 00PM TO 6 : 00 PM

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

٢٦. Young's double slit interference apparatus is immersed in a liquid of refractive index ١.٤٤. It has slit separation of ١.٥mm. The slits are illuminated by a parallel beam of light whose wavelength in air is ٦٩٠ nm. The fringe-width on a screen placed behind the plane of slits at a distance of ٠.٧٢m will be :

(١)٠.٢٣ mm (٢)٠.٣٣ mm

(٣)٠.٦٣ mm (٤)٠.٤٦ mm

Ans. (١)

Sol. $\beta = \frac{\lambda D}{d} = \frac{٦٩٠ \times ١٠^{-٩} \times ٠.٧٢}{١.٤٤ \times ١.٥ \times ١٠^{-٣}} = ٠.٢٣ \text{ mm}$

٢٧. Arrange the following in the ascending order of wavelength (λ) :

(A)Microwaves ($\lambda_١$)

(B)Ultraviolet rays ($\lambda_٢$)

(C)Infrared rays ($\lambda_٣$)

(D)X-rays ($\lambda_٤$)

Choose the most appropriate answer from the options given below :-

(١) 4, 3, 2,

(٢) 3, 4, 2,

(٣) 4, 2, 3,

(٤) 4, 3, 1,

Ans. (٣)

Sol.

γ-rays	X-rays	U.V rays	Visible rays	IR rays	Micro waves	Radio waves

$\lambda \uparrow$

٢٨. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason(R).

Assertion (A) : A electron in a certain region of uniform magnetic field is moving with constant velocity in a straight line path.

Reason (A) : The magnetic field in that region is along the direction of velocity of the electron.

In the light of the above statements, choose the correct answer from the options given below :

(١)(A) is false but (R) is true

(٢)Both (A) and (R) are true and (R) is the correct explanation of (A)

(٣)Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(٤)(A) is true but (R) is false

Ans. (٢)

Sol. $\vec{F} = q\vec{v} \times \vec{B}$

$\vec{F} = 0$,

$\vec{v} \parallel \vec{B}$

$\theta = ٠ \text{ or } ١٨٠$

٢٩. A solid sphere is rolling without slipping on a horizontal plane. The ratio of the linear kinetic energy of the centre of mass of the sphere and rotational kinetic energy is :

(١) $\frac{٢}{٥}$ (٢) $\frac{٥}{٢}$

(٣) $\frac{٣}{٤}$ (٤) $\frac{٤}{٣}$

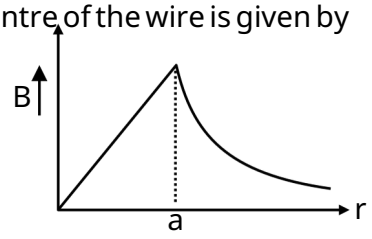
Ans. (٢)

Sol. $\frac{\text{Linear KE}}{\text{Rotational K.E}} = \frac{\frac{١}{٢}mv_{cm}^{٢}}{\frac{١}{٢}I\omega^{٢}}$

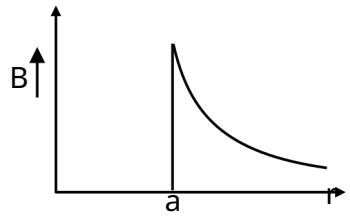
$\frac{mv_{cm}^{٢}}{\frac{٢}{٥}mR^{٢}\omega^{2}} = \frac{٥}{٢}$ $(V = \omega R)$

٣٠. A long straight wire of a circular cross-section with radius 'a' carries a steady current I. The current I is a uniformly distributed across this cross-section. The plot of magnitude of magnetic field B with distance r from the centre of the wire is given by

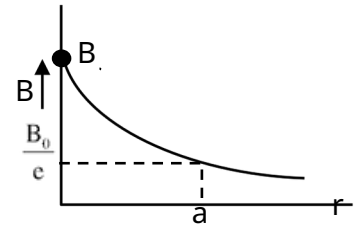
(١)



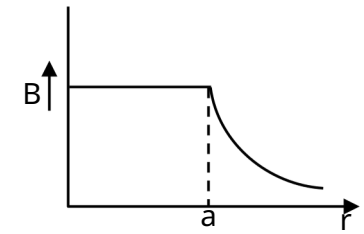
(٢)



(٣)

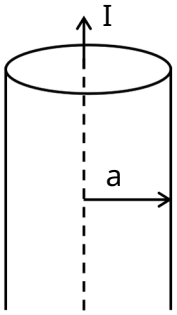


(٤)



Ans. (١)

Sol.



$B_{in} \propto \frac{J}{2}$

$B_{in} \propto r$

$B_{out} = \frac{I}{2\pi r}$

$B_{net} \propto \frac{1}{r}$

٣١. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason(R). Assertion (A) : In an insulated container, a gas is adiabatically shrunk to half of its initial volume. The temperature of the gas decreases. Reason (R) : Free expansion of an ideal gas is an irreversible and an adiabatic process. In the light of the above statement, choose the correct answer from the options given below : (١)Both (A) and (R) are true and (R) is the correct explanation of (A) (٢)(A) is true but (R) is false (٣)(A) is false but (R) is true (٤)Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

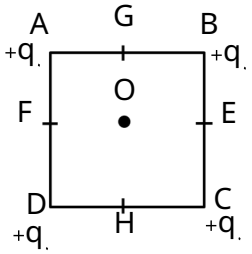
Ans. (٣)

Sol. (A) $TV^{\gamma-1} = TV^{\gamma-1}$

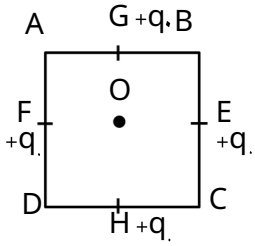
Temp increases

R)Free expansion is assumed fast, so Adiabatic

٣٢.



Configuration(١)



Configuration(٢)

In the first configuration (١) as shown in the figure, four identical charges (q) are kept at the corners A, B, C and D of square of side length 'a'. In the second configuration (٢), the same charges are shifted to mid points G, E, H and F, of the square, If $K = \frac{1}{4\pi\epsilon_0}$, the difference between the potential energies ofconfiguration (٢) and (١) is given by :

(١) $\frac{Kq^2}{a}(\sqrt{2}+2)$ (٢) $\frac{Kq^2}{a}(2\sqrt{2}+1)$

(٣) $\frac{Kq^2}{a}(2\sqrt{2}-1)$ (٤) $\frac{Kq^2}{a}(\sqrt{2}-2)$

Ans. (٤)

Sol. $U_1 = \frac{4Kq^2}{a} + \frac{2Kq^2}{\sqrt{2}a} + \frac{Kq^2}{a} = 4\sqrt{2}$

$U_2 = \frac{Kq^2}{a}\left(4 + \sqrt{2} + \frac{1}{\sqrt{2}}\right) + 2$

$\frac{Kq^2}{a}$

$U_2 - U_1 = \frac{}{a}\sqrt{3} + 2$

33. The position vector of a moving body at any instant of time is given as $\vec{r} = 5t^2\hat{i} + 5t\hat{j}$ m. The magnitude and direction of velocity at t = 2 s is,

(1) $5\sqrt{5}$ m/s, making an angle of $\tan^{-1}4$ with –ve Y axis

(2) $5\sqrt{5}$ m/s, making an angle of $\tan^{-1}4$ with +ve X axis

(3) $5\sqrt{7}$ m/s, making an angle of $\tan^{-1}4$ with –ve Y axis

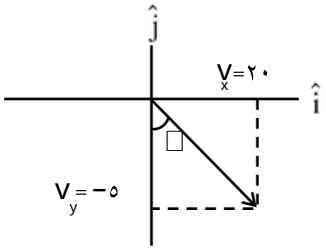
(4) $5\sqrt{7}$ m/s, making an angle of $\tan^{-1}4$ with +ve X axis

Ans. (3)

Sol. $\vec{r} = 5t\hat{i} + 5t\hat{j}$

$\vec{v} = 10t\hat{i} + 5\hat{j}$

$\vec{v} = 20\hat{i} + 5\hat{j}$ at t = 2 sec



$\tan\theta = \frac{20}{5} = 4$

$\theta = \tan^{-1}4$

From –ve Y–axis

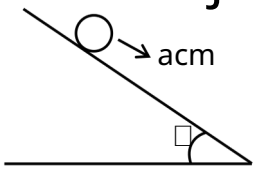
34. A solid sphere and a hollow sphere of the same mass and of same radius are rolled on an inclined plane. Let the time taken to reach the bottom by the solid sphere and the hollow sphere be t_1 and t_2 respectively, then

(1) $t_1 > t_2$ (2) $t_1 = t_2$

(3) $t_1 = 2t_2$ (4) $t_1 < t_2$

Ans. (1)

Sol.



$t = \sqrt{\frac{2\ell}{a_{cm}}}$

$a_{cm} = \frac{g\sin\theta}{1 + \frac{I_{cm}}{MR^2}}$

$a_1 = a_{cm_1} = \frac{5}{7}g\sin\theta$ ..... Solid

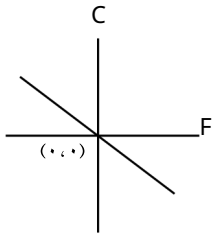
$a_2 = a_{cm_2} = \frac{3}{5}g\sin\theta$ ..... Hollow

$a_1 < a_2$

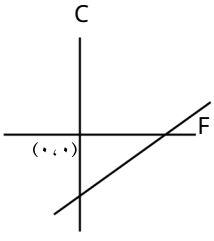
$t_1 > t_2$

35. Which of the following figure represents the relation between Celsius and Fahrenheit temperatures?

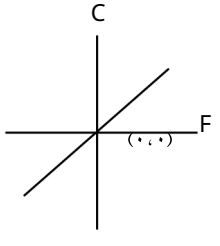
(1)



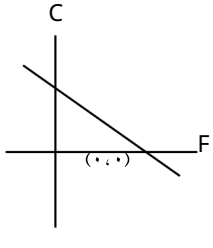
(2)



(3)

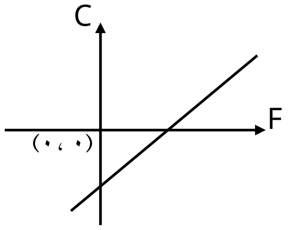


(4)

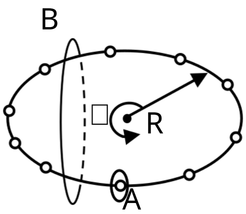


Ans. (2)

Sol. $\frac{C}{٥} □ \frac{F□٣٢}{٩}$ □ $C □ \frac{٥F}{٩} □ \frac{١٦٠}{٩}$



٣٦.



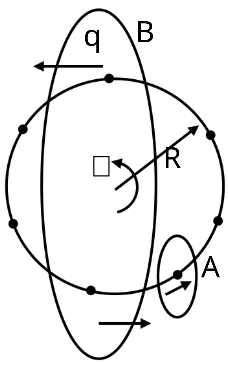
N equally spaced charges each of value q، are placed on a circle of radius R. The circle rotates about its axis with an angular velocity □ as shown in the figure. A bigger Amperian loop B encloses the whole circle where as a smaller Amperian loop A encloses a small segment. The difference between enclosed currents، $I_A - I_B$، for the given Amperian loops is

(١) $\frac{N^٢}{٢□}q□$ (٢) $\frac{٢□}{N}q□$

(٣) $\frac{N}{٢□}q□$ (٤) $\frac{N}{□}q□$

Ans. (٣)

Sol.



$I_A □ \frac{\frac{Nq}{٢□}}{□}$

$I_A □ \frac{Nq□}{٢□}$ ، I_B = ٠

$I_A □ I_B \frac{Nq□}{٢□}$

٣٧. In photoelectric effect، the stopping potential ($V_.$) v/s frequency (v) curve is plotted.

(h is the Planck's constant and □ is work function of metal)

(A) $V_.$ v/s □ is linear

(B) The slope of $V_.$ v/s □ curve = $\frac{□}{h}$

(C) h constant is related to the slope of $V_.$ v/s □ line

(D) The value of electric charge of electron is not required to determine h using the $V_.$ v/s □ curve.

(E) The work function can be estimated without knowing the value of h.

Choose the correct answer from the options given below:

(١) (A)، (B) and (C) only

(٢) (C) and (D) only

(٣) (A)، (C) and (E) only

(٤) (D) and (E) only

Ans. (٣)

Sol. hv = □ + KE_{max}

KE_{max} = $eV_.$

$V_. □ \frac{hv□□}{e}$

(A) $V_.$ v/s V is linear correct

(B) Slope

$V_. □□\frac{h}{e}□v□\frac{□}{e}$ Wrong

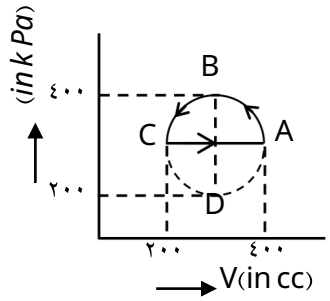
Slope $\frac{h}{e}$

(C) Correct

(D) Incorrect

(E) Correct

٣٨. The magnitude of heat exchanged by a system for the given cyclic process ABCA (as shown in figure) is (in SI unit)

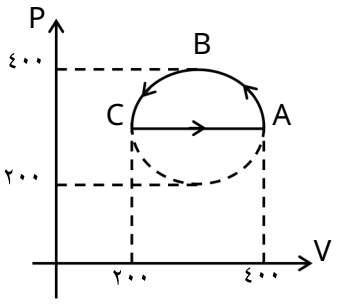


(١) ١٠ (٢) ٥

(٣) zero (٤) ٤٠

Ans. (٢)

Sol.



$W = \frac{1}{2} \times R^2$

$= \frac{1}{2} \times \left(\frac{200}{2}\right) \times 10^3 \times \left(\frac{200}{2}\right) \times 10^{-6}$

$= \frac{10}{2} = 5\,J$

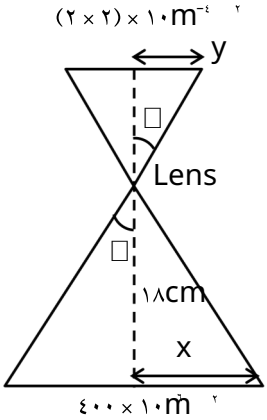
٣٩. A photograph of a landscape is captured by a drone camera at a height of ١٨ km. The size of the camera film is ٢ cm × ٢ cm and the area of the landscape photographed is ٤٠٠ km^2. The focal length of the lens in the drone camera is :

(١) ١.٨ cm (٢) ٢.٨ cm

(٣) ٢.٥ cm (٤) ٠.٩ cm

Ans. (١)

Sol.



H = ١٨ km

Size of camera film = ٢cm × ٢cm

A_{image} = ٤٠٠ km^2

$x = 200 \times 10^{-4}\,m = 2 \times 10^{-2}\,m$

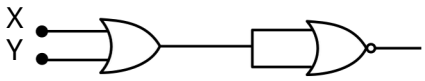
$y = 2 \times 10^{-2}\,m$

$\frac{x}{y} = 10^6 = \frac{18\,Km}{f}$

$f = 18 \times 10^{-3}\,m = 18\,mm$

$f = 1.8\,cm$

٤٠. The output of the circuit is low (zero) for :



(A) X = ٠, Y = ٠ (B) X = ٠, Y = ١

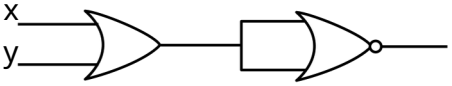
(C) X = ١, Y = ٠ (D) X = ١, Y = ١

Choose the correct answer from the options given below : (١) (A), (C) and (D) only (٢) (A), (B) and (C) only (٣) (B), (C) and (D) only (٤) (A), (B) and (D) only

(٣)

Ans.

Sol.



x

٠	y	
٠	٠	١
١	١	٠
١	٠	٠
	١	٠

٤١. The temperature of a body in air falls from ٤٠°C to ٢٤°C in ٤ minutes. The temperature of the air is ١٦°C. The temperature of the body in the next ٤ minutes will be :

(١) $\frac{١٤}{٣}$°C (٢) $\frac{٢٨}{٣}$°C

(٣) $\frac{٥٦}{٣}$°C (٤) $\frac{٤٢}{٣}$°C

Ans. (٣)

Sol. $\frac{T_1+T_2}{t}=K(T_{avg}-T_s)$

$T_1=٢٤°C$, $T_2=٤٠°C$, $t=٤$, $T_s=١٦°C$

$\frac{٤٠-٢٤}{٤}=K\left(\frac{٤٠+٢٤}{٢}-١٦\right)$

$K=\frac{٤}{١٦}=\frac{١}{٤}$

Now $\frac{٢٤-T}{٤}=K\left(\frac{T+٢٤}{٢}-١٦\right)$

$٢٤-T=\frac{T-١٦}{٢}-١٦$

$\frac{٣T}{٢}=٢٨$

$T=\frac{٥٦}{٣}C$

٤٢. The energy E and momentum p of a moving body of mass m are related by some equation. Given that c represents the speed of light, identify the correct equation.

(١) $E^2=p^4c^2+m^2c^4$ (٢) $E=p^2c^2+m^2c^4$

(٣) $E^2=p^2c^2+m^2c^4$ (٤) $E=p^2c^2+m^2c^4$

Ans. (٤)

Sol. $[E]=ML^2T^{-2}$

$[pc]=ML^2T^{-1}.LT^{-1}=ML^2T^{-2}$

$[mc^2]=ML^2T^{-2}$

$E^2=M^2L^4T^{-4}$

$E^2=p^2c^2+m^2c^4$

٤٣. A small uncharged conducting sphere is placed in contact with an identical sphere but having ٤×١٠^{-٨} C charge and then removed to a distance such that the force of repulsion between them is ٩×١٠^{-٣} N.

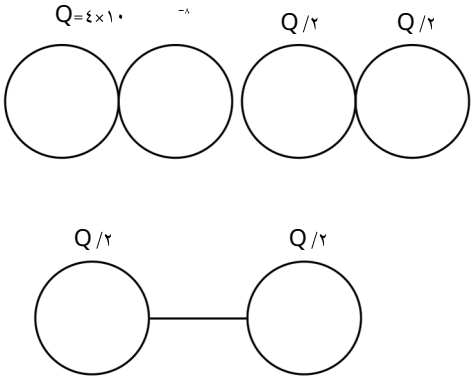
The distance between them is (Take $\frac{١}{4\pi\varepsilon_0}$ as ٩×١٠^{٩} in SI units)

(١) ٢ cm (٢) ٣ cm

(٣) ٤ cm (٤) ١ cm

Ans. (١)

Sol.



$F=\frac{k(Q/2)(Q/2)}{r^2}$

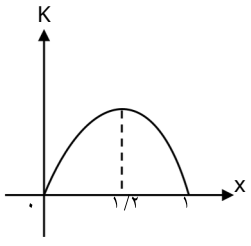
$٩\times١٠^{-٣}=\frac{٩\times١٠^{٩}\times(٤\times١٠^{-٨})\times٤\times١٠^{-٨}}{٤r^2}$

$r^2=\frac{٩\times١٦\times١٠^{-١٦}}{٤\times٩\times١٠^{-٣}}=٤\times١٠^{-٤}$

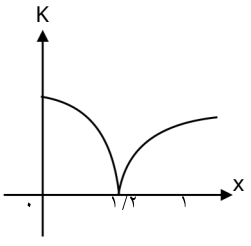
$r=٢\times١٠^{-٢}m=٢$ cm

٤٤. A particle oscillates along the x-axis according to the law, $x(t)=x\sin\left(\frac{2\pi t}{2}\right)$ where $x=١$ m. The kinetic energy (K) of the particle as a function of x is correctly represented by the graph.

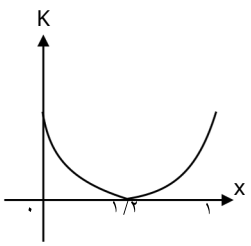
(١)



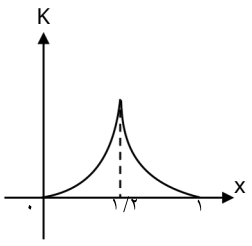
(٢)



(٣)



(٤)



Ans. (١)

Sol. $x = x\sin\frac{t}{T} = x\left(\frac{1-\cos t}{2}\right)$

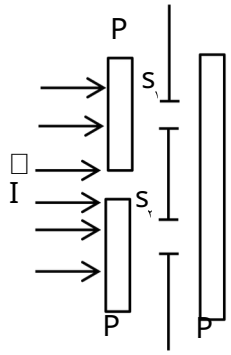
$x = \frac{x}{2}(1-\cos t)$

where $x = 1$

$x = \frac{1}{2}(1-\cos t)$

Particle is oscillating between x = 0 to x = 1 In a

45. Young's double slit experiment, three polarizers are kept as shown in the figure. The transmission axes of P_1 and P_2 are orthogonal to each other. The polarizer P_3 covers both the slits with its transmission axis at 45° to those of P_1 and P_2. An unpolarized light of wavelength λ and intensity I_0 is incident on P_1 and P_2. The intensity at a point after P_3 where the path difference between the light waves from s_1 and s_2 is $\frac{\lambda}{4}$

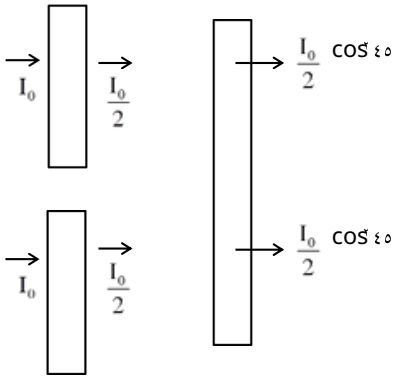


(1) $\frac{I_0}{2}$ (2) $\frac{I_0}{4}$

(3) I_0 (4) $\frac{I_0}{3}$

Ans. (3)

Sol.



(Unpolerised light) (Polerised light)

after passing through third poleriser, Intensity of both the waves must be $\frac{I_0}{4}$.

now, at a point where path diff is $\frac{\lambda}{4}$, phase difference

$\Delta\phi = 2K\left(\frac{\Delta x}{\lambda}\right)2\pi$

$I_{res} = \sqrt{\left(\frac{I_0}{4}\right)^2 + \left(\frac{I_0}{4}\right)^2 + 2\left(\frac{I_0}{4}\right)^2\cos\frac{\pi}{2}}$

$= \frac{I_0}{4}$

46. A tightly wound long solenoid carries a current of 1.5 A. An electron is executing uniform circular motion inside the solenoid with a time period of 75ns. The number of turns per metre in the solenoid is_____. Take mass of electron $m_e = 9\times10^{-31}$ kg, charge of electron $|q_e| = 1.6\times10^{-19}$ C, $\mu_0 = 4\pi\times10^{-7}\frac{N}{A^2}$, 1 ns = 10^{-9} s

Ans. (250)

Sol. Since time period of a revolving charge is $\frac{2\pi m}{qB}$

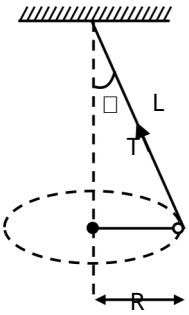
Where B = magnetic field

due to a solenoid = μnI

$T = \frac{2\pi m}{q(\mu_0 nI)}$

$75\times10^{-9} = \frac{(2\pi)(9\times10^{-31})}{1.6\times10^{-19}\times4\pi\times10^{-7}\times n\times1.5}$

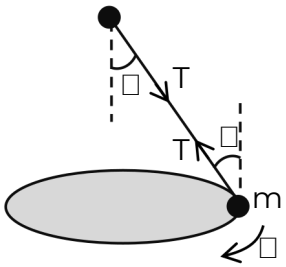
N = 250

٤٧. 

A string of length L is fixed at one end and carries a mass of M at the other end. The mass makes $\frac{3}{\pi}$ rotations per second about the vertical axis passing through end of the string as shown. The tension in the string is _____ ML.

Ans. (٣٦)

Sol.



$T\cos\theta = mg$ ...(١)

$T\sin\theta = M\omega^2 R$ ...(٢)

Using equation (٢)

$T\sin\theta = M\omega^2(L\sin\theta)$

$T = ML\omega^2 = M\left(2\pi\frac{3}{2}\right)^2 L$

$T = ٣٦\,ML$

٤٨. The ratio of the power of a light source S to that the light source S_2 is ٢. S_1 is emitting ٢×١٠^{١٥} photons per second at ٦٠٠ nm. If the wavelength of the source S_2 is ٣٠٠ nm, then the number of photons per second emitted by S is _____ $\times١٠^{١٤}$.

Ans. (٥)

Sol. Since power emitting by a source is given as

$$\frac{\text{Total energy emitted}}{\text{time}}$$

$$\frac{(E1\text{ photon})\times\text{Number of photons}(N)}{t}$$

$P_1 = (E)n_1$

$$\frac{P_1}{P_2} = \frac{(E1)n1}{(E2)n2} = \frac{\frac{hC}{\lambda_1}n_1}{\frac{hC}{\lambda_2}n_2}$$

$$\frac{P_1}{P_2} = \frac{\lambda_2 n_1}{\lambda_1 n_2}$$

Substituting the given values

$$2 = \frac{300}{600}\cdot\frac{2\times10^{15}}{n_2}$$

$$n_2 = \frac{1}{2}\times10^{15} = ٥\times١٠^{١٤}\ \text{Photon/sec}$$

٤٩. The increase in pressure required to decrease the volume of a water sample by ٠.٢% is $P\times١٠^{٧}\ Nm^{-٢}$. Bulk modulus of water is $٢.١٥\times١٠^{٩}\ Nm^{-٢}$. The value of P is _____.

Ans. (٤٣)

Sol. Since bulk modulus is given as

$$B = -\frac{\Delta P}{\frac{\Delta V}{V}}$$

$$2.15\times10^{9} = -\frac{\Delta P}{\frac{0.2}{100}}$$

$\Delta P = ٢.١٥\times١٠^{٩}\times٢\times١٠^{-٣}$

$= ٤.٣\times١٠^{٦} = ٤٣\times١٠^{٥}\ N/m^{٢}$

٥٠. Acceleration due to gravity on the surface of earth is 'g'. If the diameter of earth is reduced to one third of its original value and mass remains unchanged, then the acceleration due to gravity on the surface of the earth is _____g.

Ans. (٩)

Sol. $\because$ acceleration due to gravity on surface is given by

$$g = \frac{GM}{R_e^2}$$

Now since diameter is reduced to $١/٣^{rd}$, radius also reduces to $١/٣$, keeping mass constant

New value of acceleration due to gravity on Earth's surface is

$$g' = \frac{GM}{\left(\frac{R_e}{3}\right)^2} = 9\frac{GM_e}{R_e^2} = 9g$$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON FRIDAY24 th JANUARY2025) TIME : 3 : 00PM TO 6 : 00 PM

CHEMISTRY

TEST PAPER WITH SOLUTIONS

SECTION-A

٥١. Based on the data given below :

$E^{\circ}_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33\,V$ $E^{\circ}_{Cl_2/Cl^-} = 1.36\,V$

$E^{\circ}_{MnO_4^-/Mn^{2+}} = 1.51\,V$ $E^{\circ}_{Cr^{3+}/Cr} = -0.74\,V$

the strongest reducing agent is :

(١) Mn^{2+} (٢) Cr

(٣) MnO_4^- (٤) Cl^-

Ans. (٢)

Sol. For strongest reducing agent

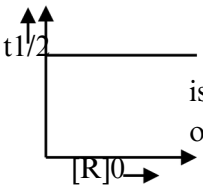
Reduction potential should be lowest

Hence Cr is the strongest reducing agent.

Options (٢)

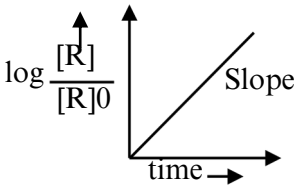
٥٢. Given below are two statements :

Statement(I) :



is valid for first order reaction.

Statement(II) :



is valid for first order reaction

In the light of the above statements, choose the correct answer from the options given below :

(١) Both Statement I and Statement II are false

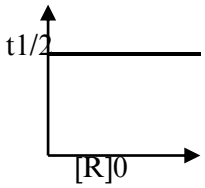
(٢) Statement I is false but Statement II is true

(٣) Both Statement I and Statement II are true

(٤) Statement I is true but Statement II is false

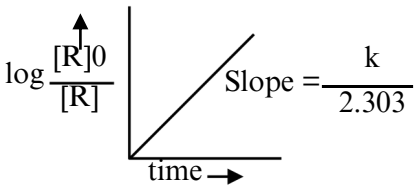
Ans. (٤)

Sol. For first order reaction $t_{1/2} = \frac{\ell n 2}{k}$



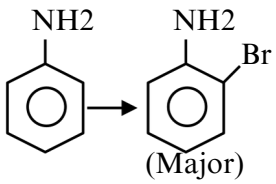
For first order reaction

$\log \frac{[R]_0}{[R]} = \frac{k}{2.303} t$



Options (٤)

٥٣. For reaction



The correct order of set of reagents for the above conversion is :

(١) $Br_2, FeBr_3, H_2O, NaOH$

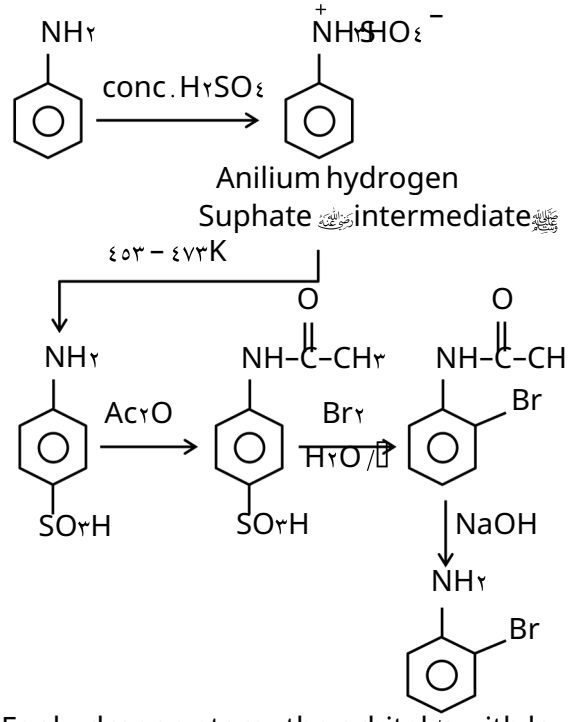
(٢) $H_2SO_4, AcO, Br_2, H_2O, NaOH$

(٣) $AcO_2, Br_2, H_2O, NaOH$

(٤) $AcO_2, H_2SO_4, Br_2, NaOH$

Ans. (٢)

Sol.



54. For hydrogen atom, the orbital/s with lowest energy is/are :

(A) 4s (B) 3px

(C) $3d_{x^2-y^2}$ (D) $3d_{z^2}$

(E) $4p_z$

Choose the correct answer from the options given below :

(1) (A) and (E) only (2) (B) only

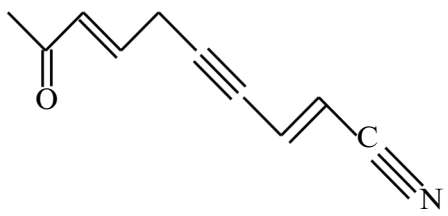
(3) (A) only (4) (B), (C) and (D) only

Ans. (4)

Sol. In hydrogen atom the orbitals in a shell are degenerate means energy depends only on 'n'

$E_{3px} = E_{3d_{x^2-y^2}} = E_{3d_{z^2}}$

55. Inthe given structure, number of sp and sp^2 hybridized carbon atoms present respectively are :

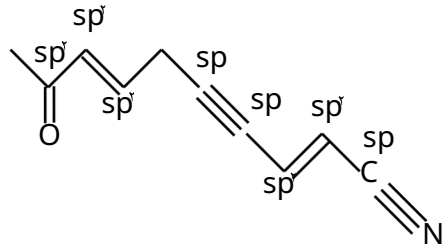


(1) 3 and 6 (2) 3 and 5

(3) 4 and 6 (4) 4 and 5

Ans. (2)

Sol.



Number of sp and sp^2 hybridised carbon atom are 3 and 5.

56. Which of the following mixing of 1M base and 1M acid leads to the largest increase in temperature?

(1) 30 mL HCl and 30 mL NaOH

(2) 30 mL CH_3COOH and 30 mL NaOH

(3) 50 mL HCl and 20 mL NaOH

(4) 45 mL CH_3COOH and 25 mL NaOH

Ans. (1)

Sol. Higher the number of milli moles of acid or base reacted higher will be temperature rise.

Option (4) nacid or nbase reacted = 30 m mol

Option (2) nacid or nbase reacted = 30 m mol but less energy will be released by neutralisation reaction of weak acid hence option (2) can not be correct.

Option (3) 20 m mol

Option (4) 25 m mol

Hence Correct Option (1)

57. Given below are two statements :

Statement (I) : Experimentally determined oxygen-oxygen bond lengths in the O_3 are found to be same and the bond length is greater than that of a O = O (double bond) but less than that of a single (O – O) bond.

Statement (II) : The strong lone pair-lone pair repulsion between oxygen atoms is solely responsible for the fact that the bond length in ozone is smaller than that of a double bond (O=O) but more than that of a single bond (O – O).

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. (1)

Sol. Due to resonance bond length is identical in ozone. Therefore statement I is true and statement II is false

৫৮. Find the compound 'A' from the following reaction sequences.

A $\xrightarrow{\text{aqua regia}}$ B $\xrightarrow[\text{2. AcOH}]{\text{1. KNO}_2\text{, NH}_4\text{OH}}$ yellow ppt

(১) ZnS (২) CoS

(৩) MnS (৪) Nis

Ans. (২)

Sol. $CoS \xrightarrow{\text{aqua regia}} Co^{2+} \xrightarrow[\text{2. AcOH}]{\text{1. KNO}_2\text{, NH}_4\text{OH}}$

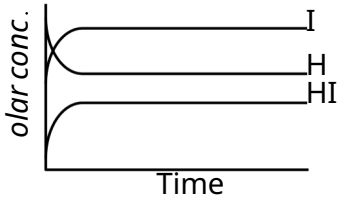
$K_3Co(NO_2)_6$ yellow

৫৯. For the reaction,

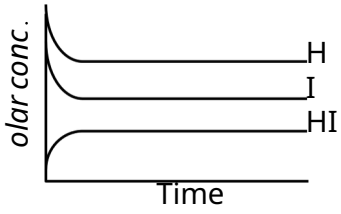
$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$

Attainment of equilibrium is predicted correctly by :

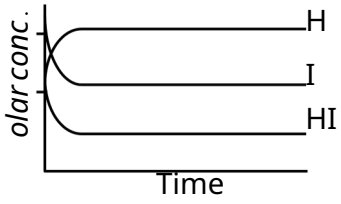
(১)



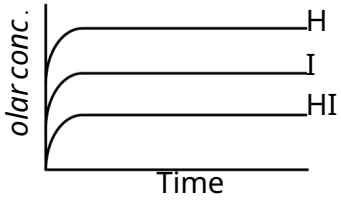
(২)



(৩)



(৪)



Ans. (২)

Sol. $H_2 + I_2 \rightleftharpoons 2HI$

Concentration of H_2 and I_2 decreases untill equilibrium condition and concentration of HI increases till equilibrium condition and after equilibrium concentration of all the reactant and products remain constant.

Correct option (২)

৬০. Match List-I with List-II.

List-I (Transition metal ion)	List-II (Spin only magnetic moment (B.M.))
(A) Ti^{3+}	(I) ৩.৮৭
(B) V	(II) ০.০০
(C) Ni	(III) ১.৭৩
(D) Sc	(IV) ২.৮৪

Choose the correct answer from the options given below :

(১) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(২) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(৩) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(৪) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

Ans. (২)

Sol. $Sc^{+3} = 3d^0$ spin = ০, spin = ৩.৮৭ B.M.

$V^{+2} = 3d^3$ spin = ২.৮৪ B.M.

$Ni^{+2} = 3d^8$

$Ti^{+3} = 3d^1$ spin = ১.৭৩ B.M.

৬১. The elemental composition of a compound is ৫৪.২%, C, ৯.২% H and ৩৬.৬% O. If the molar mass of the compound is ১৩২ g mol^{-1}, the molecular formula of the compound is :

Given : The relative atomic mass of C : H : O = ১২ : ১ : ১৬

(১) $C_6H_9O_3$ (২) $C_6H_{12}O_6$

(৩) $C_6H_{12}O_3$ (৪) $C_6H_8O_2$

Ans. (৩)

Sol.

C	:	H	:	O
৫৪.২ / ১২	:	৯.২ / ৯.২	:	৩৬.৬ / ১৬
৪.৫১৬	:		:	২.২৮৭
৪.৫১৬ / ২.২৮৭		৯.২ / ২.২৮৭		২.২৮৭ / ২.২৮৭
১.৯৭		৪.০২		১

C_2H_4O Empirical formula E.F. mass = ২৪ + ৪ + ১৬ = ৪৪

and molar mass = ১৩২ = $(C_2H_4O)_3$

Hence molecular formula = $C_6H_{12}O_3$

Correct Option (৩)

٦٢. When Ethane-١,٢-diamine is added progressively to an aqueous solution of Nickel (II) chloride, the sequence of colour change observed will be :
(١) Pale Blue → Blue → Green → Violet
(٢) Pale Blue → Blue → Violet → Green
(٣) Green → Pale Blue → Blue → Violet
(٤) Violet → Blue → Pale Blue → Green

Ans. (٣)

Sol.

$[Ni(H_2O)_6]^{2+}(aq) + en(aq) \rightarrow [Ni(H_2O)_4(en)]^{2+}(aq) + 2H_2O$

→ Green

$[Ni(H_2O)_4(en)]^{2+}(aq) + en(aq) \rightarrow [Ni(H_2O)_2(en)_2]^{2+}(aq) + 2H_2O$

Pale Blue

$[Ni(H_2O)_2(en)_2]^{2+}(aq) + en(aq) \rightarrow [Ni(en)_3]^{2+}(aq) + 2H_2O$

→ Blue / purple

٦٣. The conditions and consequence that favours the t_{2g}^3, e_g^0 configuration in a metal complex are :
(١) weak field ligand, high spin complex
(٢) strong field ligand, high spin complex
(٣) strong field ligand, low spin complex
(٤) weak field ligand, low spin complex

Ans. (١)

Sol. For d^4

If ligand is SFL : $t_{2g}^4 e_g^0$ (Low spin)

If ligand is WFL : $t_{2g}^3 e_g^1$ (High spin)

٦٤. Identify correct statement/s :
(A) —OCH_3 and —$NHCOCH_3$ are activating group
(B) —CN and —OH are meta directing group
(C) —CN and —SO_3H are meta directing group
(D) Activating groups act as ortho – and para directing groups
(E) Halides are activating groups
Choose the correct answer from the options given below :
(١) (A), (C) and (D) only
(٢) (A), (B) and (E) only
(٣) (A) only
(٤) (A) and (C) only

Ans. (١)

Sol. (B) –CN is meta directing But –OH is ortho / para directing.
(E) Halides are deactivating groups.

٦٥. Given below are two statements :
Statement (I) : The first ionization energy of Pb is greater than that of Sn
Statement (II) : The first ionization energy of Ge is greater than that of Si
In the light of the above statements, choose the correct answer from the options given below :
(١) Statement I is true but Statement II is false
(٢) Both Statement I and Statement II are false
(٣) Statement I is false but Statement II is true
(٤) Both Statement I and Statement II are true

Ans. (١)

Sol. Order of I.E. (in KJ/mol) :

C < Si < Ge < Sn > Pb

١٠٨٦ ٧٨٦ ٧٦١ ٧٠٨ ٧١٥

٦٦. $S(g) + \frac{3}{2}O_2(g) \rightarrow SO_3(g) + x$ kcal

$SO_2(g) + \frac{1}{2}O_2(g) \rightarrow SO_3(g) + y$ kcal

The heat of formation of $SO_2(g)$ is given by :
(١) $\frac{2x}{y}$ kcal
(٢) $y - 2x$ kcal
(٣) $2x + y$ kcal
(٤) $x + y$ kcal

Ans. (٢)

Sol. $S_{(g)} + O_{2(g)} \rightarrow SO_{2(g)}$ $\Delta H = -y$

$\Delta H_r = (\Delta H_f)_{SO_3} - (\Delta H_f)_{SO_2}$

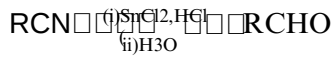
$-y = 2x - (\Delta H_f)_{SO_2}$

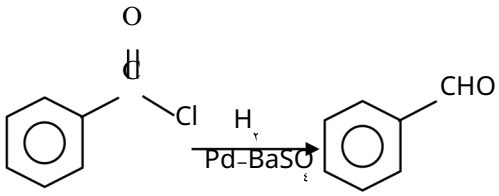
$(\Delta H_f)_{SO_2} = y - 2x$

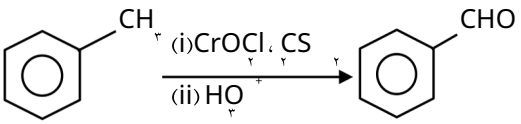
Option (٢)

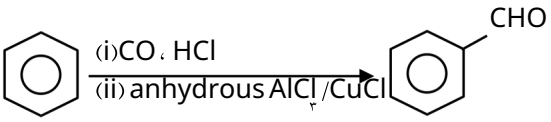
६७. Match List-I with List-II

List-I

(A) 

(B) 

(C) 

(D) 

List-II

(I) Etard reaction

(II) Gatterman –Koch reaction

(III) Rosenmund reduction

(IV) Stephen reaction

Choose the correct answer from the options given below :

(१) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

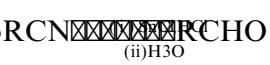
(२) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(३) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

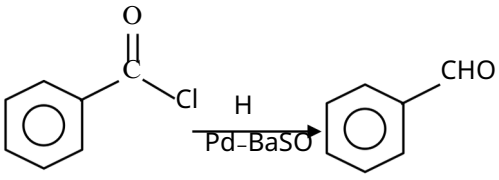
(४) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Ans. (१)

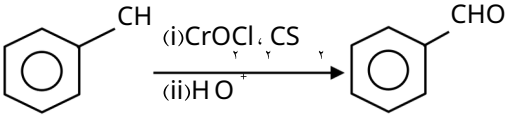
Sol. List-I

(A) 

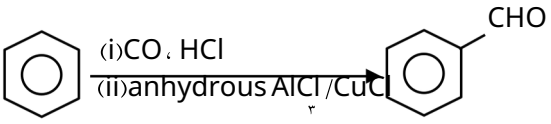
Stephen reaction

(B) 

Rosenmund reduction

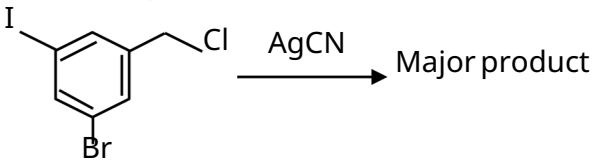
(C) 

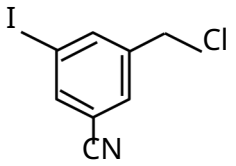
Etard reaction

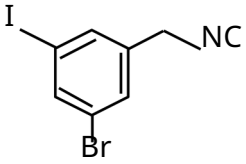
(D) 

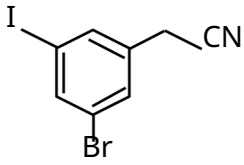
Gatterman –Koch reaction

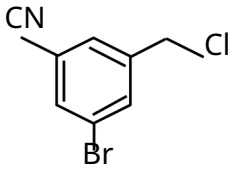
६८. The structure of the major product formed in the following reaction is :



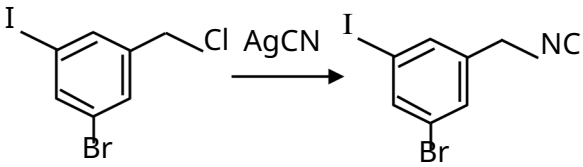
(१) 

(२) 

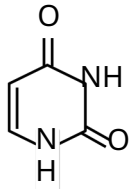
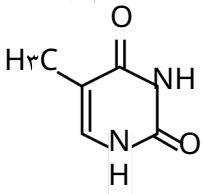
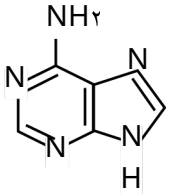
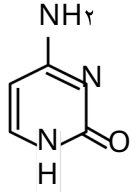
(३) 

(४) 

Ans. (२)

Sol. 

६९. Match List-I with List-II.

List-I		List-II	
(A)	Adenine	(I)	
(B)	Cytosine	(II)	
(C)	Thymine	(III)	
(D)	Uracil	(IV)	

Choose the correct answer from the options given below :

(१) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

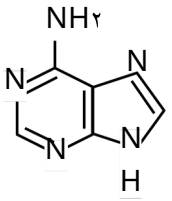
(२) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

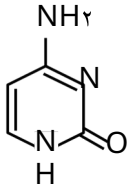
(३) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

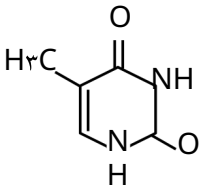
(४) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

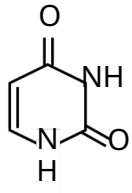
Ans. (१)

Sol.

(A) Adenine (III) 

(B) Cytosine (IV) 

(C) Thymine (II) 

(D) Uracil (I) 

٧٠. The successive ٥ ionisation energies of an element are ٨٠٠، ٢٤٢٧، ٣٦٥٨، ٢٥٠٢٤ and ٣٢٨٢٤ kJ/mol، respectively. By using the above values predict the group in which the above element is present : (١)Group ٢ (٢)Group ١٣ (٣)Group ٤ (٤)Group ١٤

Ans. (٢)

Sol. The IE٤ is suddenly very high therefore element must have ٣ valence ē(s) and it belong to group ١٣

SECTION-B

٧١. The observed and normal masses of compound MX_2 are ٦٥.٦ and ١٦٤ respectively. The percent degree of ionisation of MX_2 is _____ ٪. (Nearest integer)

Ans.(٧٥)

Sol. $MX_2 \rightleftharpoons M^{2+} + 2X^-$

$i = \frac{\text{normal molar mass}}{\text{observed molar mass}}$

$i = \frac{١٦٤}{٦٥.٦}$

$1 + (3-1)\alpha = \frac{١٦٤}{٦٥.٦}$

$2\alpha = \frac{٩٨.٤}{٦٥.٦}$

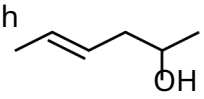
$\alpha = ٠.٧٥$

percent dissociation = ٧٥٪

Ans. ٧٥

٧٢. The possible number of stereoisomers for ٥-phenylpent-٤-en-٢-ol is _____.

Ans. (٤)

Sol. 

n (stereogenic unit) = ٢، 2^2 = ٤ stereoisomers are possible.

٧٣. Consider a complex reaction taking place in three steps with rate constants k_1، k_2 and k_3 respectively. The overall rate constant k is given by the expression $k = \sqrt{\frac{k_1 k_3}{k_2}}$. If the activation energies of the three steps are ٦٠، ٣٠ and ١٠ kJ mol^{-1} respectively، then the overall energy of activation in kJ mol^{-1} is _____. (Nearest integer)

Ans. (٢٠)

Sol. $K = \sqrt{\frac{K_1 K_3}{K_2}}$

$A.e^{-E_a/RT} = \sqrt{\frac{A_1 e^{-E_{a_1}/RT} \cdot A_3 e^{-E_{a_3}/RT}}{A_2 e^{-E_{a_2}/RT}}}$

By comparinig exponential term

$\frac{E_a}{RT} = \frac{1}{2}\left(\frac{E_{a_1}}{RT} + \frac{E_{a_3}}{RT} - \frac{E_{a_2}}{RT}\right)$

$E_a = (E_{a_1} + E_{a_3} - E_{a_2})/2$

Ea = (٦٠ + ١٠ − ٣٠)/٢ = ٢٠ kJ mol^{-1}

Ans. ٢٠

٧٤. The hydrocarbon (X) with molar mass ٨٠ g mol^{-1} and ٩٠٪ carbon has _____ degree of unsaturation.

Ans. (٣)

Sol. Mass of carbon $= \frac{80 \times 90}{100} = 72gm$

Number of C-atoms $\frac{72}{12} = 6$

Mass of hydrogen $= \frac{80 \times 10}{800} = 8gm$

Number of H-atoms $= 8$

So molecular formula C_6H_8

D.U. $= ٦ + ١ - ٨/٢ = ٧ - ٤ = ٣$

٧٥. In Carius method of estimation of halogen, ٠.٢٥ g of an organic compound gave ٠.١٥ g of silver bromide (AgBr). The percentage of Bromine in the organic compound is _____ $\times 10^{-1}$ % (Nearest integer).

(Given : Molar mass of Ag is ١٠٨ and Br is ٨٠ g mol^{-1})

Ans. (٢٥٥)

Sol. $\%\ Bromine = \frac{\text{Molar Mass of Bromine}}{\text{Molar Mass of Silver bromide}} \times \frac{\text{Weight of AgBr}}{\text{Weight of sample}} \times 100$

$= \frac{80}{188} \times \frac{0.165}{0.25} \times 100$

$= \frac{4800}{188} = ٢٥.٥٣ = ٢٥٥ \times 10^{-1}$