JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025) TIME : 3:00 PM TO 6:00 PM

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

١. If in the expansion of $(١ + x)^p (١ - x)^q$, the coefficients of x and $x^٢$ are ١ and −٢, respectively, then $p^٢ + q^٢$ is equal to :

(١) ٨ (٢) ١٨

(٣) ١٣ (٤) ٢٠

Ans. (٣)

Sol. $(١+x)^p(١-x)^q = ({}^pC_٠ + {}^pC_١x + {}^pC_٢x^٢ + \ldots)({}^qC_٠ - {}^qC_١x + {}^qC_٢x^٢ - \ldots)$

coff of $x \Rightarrow {}^pC_٠ \cdot (-{}^qC_١) + {}^pC_١ \cdot {}^qC_٠ = ١$

$p - q = ١$

coff of $x^٢ \Rightarrow {}^pC_٠ \cdot {}^qC_٢ - {}^pC_١ \cdot {}^qC_١ + {}^pC_٢ \cdot {}^qC_٠ = -2$

$\frac{q(q-1)}{٢} - pq + \frac{p(p-١)}{٢} = -2$

$q^٢ - q - ٢pq + p^٢ - p = -٤$

$(p - q)^٢ - (p + q) = -٤$

$p + q = ٥$

$p = ٣$

$q = ٢$

so $p^٢ + q^٢ = ١٣$

٢. Let $A = \{(x, y) \in R \times R : |x + y| \geq ٣\}$ and $B = \{(x, y) \in R \times R : |x| + |y| \leq ٣\}$.

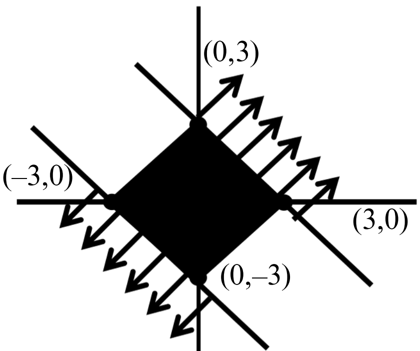
If $C = \{(x, y) \in A \cap B : x = ٠ \text{ or } y = ٠\}$, then $\sum_{(x,y)\in C} |x + y|$ is :

(١) ١٥ (٢) ١٨

(٣) ٢٤ (٤) ١٢

Ans. (٤)

Sol.



$C = \{(٣, ٠), (-٣, ٠), (٠, ٣), (٠, -٣)\}$

$\sum |x + y| = ١٢$

٣. The system of equations $x + y + z = ٦$, $x + ٢y + ٥z = ٩$, $x + ٥y + \lambda z = \mu$, has no solution if

(١) $\lambda = ١٧, \mu \neq ١٨$

(٢) $\lambda \neq ١٧, \mu \neq ١٨$

(٣) $\lambda = ١٥, \mu \neq ١٧$

(٤) $\lambda = ١٧, \mu = ١٨$

Ans. (١)

Sol. $D = \begin{vmatrix} ١ & ١ & ١ \\ ١ & ٢ & ٥ \\ ١ & ٥ & \lambda \end{vmatrix} = 0$

$\lambda = ١٧$

$D_z = \begin{vmatrix} ١ & ١ & ٦ \\ ١ & ٢ & ٩ \\ ١ & ٥ & \mu \end{vmatrix} \neq ٠$

$\mu \neq ١٨$

٤. Let $\int x^٣ \sin x\, dx = g(x) + C$, where C is the constant of integration. If $٨\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma$, $\alpha, \beta, \gamma \in Z$. Then $\alpha + \beta - \gamma$ equals :

(١) ٥٥ (٢) ٤٧

(٣) ٤٨ (٤) ٦٢

Ans. (١)

Sol. $\int x^٣ \sin x = -x^٣ \cos x + \int ٣x^٢ \cos x dx$

$= -x^٣ \cos x + ٣x^٢ \sin x - \int ٦x \sin x dx$

$= -x^٣ \cos x + ٣x^٢ \sin x + ٦x \cos x - ٦ \sin x + c$

So $g(x) = -x^٣ \cos x + ٣x^٢ \sin x + ٦x \cos x - ٦ \sin x$

$g\left(\frac{\pi}{٢}\right) = \frac{3\pi^٢}{٤} - 6$

$g'(x) = x^٣ \sin x$

$g'\left(\frac{\pi}{2}\right) = \frac{\pi^٣}{٨}$

$٨\left(g\left(\frac{\pi}{٢}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^٣ + 6\pi^2 - ٤٨$

So $\alpha + \beta - \gamma = ٥٥$

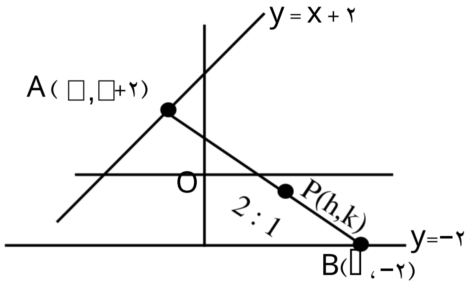
٥. A rod of length eight units moves such that its ends A and B always lie on the lines x – y + ٢ = ٠ and y + ٢ = ٠, respectively. If the locus of the point P, that divides the rod AB internally in the ratio ٢ : ١ is ٩(x + □y + □xy + □x + ٢٨y) – ٧٦ = ٠, then □ – □ – □ is equal to :

(١) ٢٤ (٢) ٢٣

(٣) ٢١ (٤) ٢٢

Ans. (٢)

Sol.



h □ ٣□ □ □ / ٣

k □ □٤ □ □ □ 2 / ٣

□= ٣k + ٢

٢□ = ٣h – a = ٣h – ٣k – ٢

so AB = ٨

[illegible]

٩ ٣k□ h □ 2□□ ٤ □k□٢ □□ □٤□٤

٩(x² + ١٣y – ٦xy – ٤x + ٢٨y) = ٧٦

□ □ □ □ □١٣ □٦ □4 □٢٣

٦. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point (١, ٤, ٠) along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :

(١) $\sqrt{17}$ (٢) $\sqrt{14}$

(٣) $\sqrt{15}$ (٤) $\sqrt{13}$

Ans. (٢)

Sol. Let the parallel line is

$\frac{x-١}{١} = \frac{y-٤}{٢} = \frac{z-٠}{٣}$

so their point of intersection is

(□+١, ٢□+٤, ٣□) = (٢t+ ٢, ٣t + ٦, ٤t + ٣)

□ = ٢t+١

٢□+٤ = ٣t+٦ □ t = ٠

so POI is (٢,٦,٣)

so distance □ $\sqrt{(2-1)^2+(6-4)^2+(3-0)^2} = \sqrt{١٤}$

٧. Let the point A divide the line segment joining the points P(−١, −١, ٢) and Q(٥, ٥, ١٠) internally in the ratio r : ١ (r > ٠). If O is the origin and $(\overrightarrow{OQ}\cdot\overrightarrow{OA}) - \frac{1}{5}|\overrightarrow{OP}\times\overrightarrow{OA}|^2 = ١٠$, then the value of r is :

(١) ١٤ (٢) ٣

(٣) $\sqrt{7}$ (٤) ٧

Ans. (٤)

Sol. $A\left(\frac{٥r-١}{r+١}, \frac{٥r-١}{r+١}, \frac{١٠r+٢}{r+١}\right)$

$\overrightarrow{OQ}\cdot\overrightarrow{OA} - \frac{|\overrightarrow{OP}\times\overrightarrow{OA}|^٢}{٥} = ١٠$ . . . .(١)

$\overrightarrow{OQ}\cdot\overrightarrow{OA} = \frac{١٠}{r+١}(١٥r+١)$

$|\overrightarrow{OP}\times\overrightarrow{OA}|^٢ = \frac{r^٢}{(r+١)^٢}(٨٠٠)$

so by equation (١)

$\frac{١٠}{r+١}(١٥r+1) - \frac{1}{٥}\frac{r^٢(٨٠٠)}{(r+١)^٢} = ١٠$

٢r² – ١٤r = ٠

r = ٧, r ≠ ٠

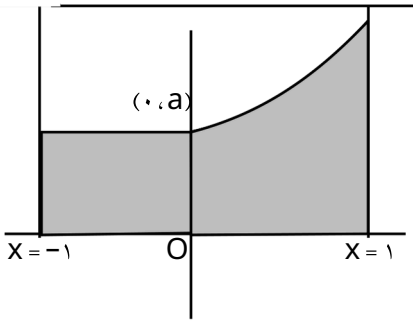
٨. If the area of the region {(x, y) : –١ ≤ x ≤ ١, ٠ ≤ y ≤ a + e^{|x|} – e^{–x}, a > ٠} is $\frac{e^2+8e+1}{e}$, then the value of a is :

(١) ٧ (٢) ٦

(٣) ٨ (٤) ٥

Ans. (٤)

Sol.



required area is $\int_{-1}^{0} \left(a - e^{x} - e^{-x}\right) dx$

$a - a e^{x} + e^{-x} \Big|_{-1}^{0}$

$2a + e - 1 + e^{-1} = e + 8 + \frac{1}{e}$

$2a = 10 \Rightarrow a = 5$

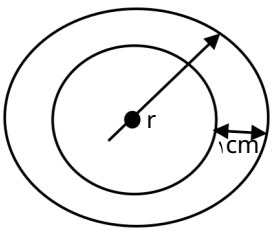
9. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

(1) 225 π (2) 128 π

(3) 196 π (4) 256 π

Ans. (4)

Sol



$V = \frac{4}{3}\pi r^3$

$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$

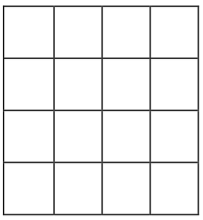
$81 = 4\pi r^2 \left(\frac{1}{4\pi}\right)$

$r^2 = 81$

$r = 9$

surface area of chocolate $= 4\pi(r-1)^2 = 256\pi$

10. A board has 16 squares as shown in the figure :



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

(1) $\frac{4}{5}$ (2) $\frac{7}{10}$

(3) $\frac{3}{5}$ (4) $\frac{23}{30}$

Ans. (1)

Sol. Total ways for selecting any two squares $= {}^{16}C_2$

$= 120$

Total ways for selecting common side squares

$= \underbrace{3+4}_{\text{Horizontal side}} + \underbrace{3+4}_{\text{vertical side}}$

$= 24$

so required probability

$= 1 - \frac{24}{120}$

$= \frac{4}{5}$

11. Let $x = x(y)$ be the solution of the differential equation $y\left(x + y\sin\left(\frac{x}{y}\right)\right)dy = \left(y\sin\left(\frac{x}{y}\right) - x\right)dy$, $y > 0$ and $x(1) = \frac{\pi}{2}$.

Then $\cos(x(2))$ is equal to :

(1) $1 - 2(\log_e 2)^2$ (2) $2(\log_e 2)^2 - 1$

(3) $2(\log_e 2) - 1$ (4) $1 - 2(\log_e 2)$

Ans. (2)

Sol. $ydy = (xdy - ydx)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y} = \frac{xdy - ydx}{y^2}\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y} = \sin\left(\frac{x}{y}\right)d\left(\frac{x}{y}\right)$

$-\ln y = \cos\frac{x}{y} + C$

$x(1) = \frac{\pi}{2} \Rightarrow 0 = \cos\frac{\pi}{2} + C \Rightarrow C = 0$

$\ln y = \cos\frac{x}{y}$

but $y = 2 \Rightarrow \cos\frac{x}{2} = \ln 2$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$= 2(\ln 2)^2 - 1$

12. Let the range of the function

$f(x) = 6 + 16\cos x \cdot \cos\left(\frac{\pi}{3} + x\right)\cdot\cos\left(\frac{\pi}{3} - x\right)\cdot\sin 3x \cdot \cos 6x$, $x \in R$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

(1) 11 (2) 8

(3) 10 (4) 9

Ans. (1)

Sol. $f(x) = 6 + 16\left(\frac{1}{4}\cos 3x\right)\sin 3x \cdot \cos 6x$

$= 6 + 4\cos 3x \sin 3x \cos 6x$

$= 6 + \sin 12x$

Range of $f(x)$ is $[5, 7]$

$(\alpha, \beta) \equiv (5, 7)$

distance $= \left|\frac{15 - 28 - 12}{5}\right| = 11$

13. Let the shortest distance from $(a, 0)$, $a < 0$, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point $(a, 0)$ and the focus of the parabola, and having its centre on the axis of the parabola is :

(1) $x^2 + y^2 - 6x + 5 = 0$

(2) $x^2 + y^2 - 4x + 3 = 0$

(3) $x^2 + y^2 - 10x + 9 = 0$

(4) $x^2 + y^2 - 8x + 7 = 0$

Ans. (1)

Sol. Normal at P

$y + tx = 2t + t^3$

$(a, 0)$

$at = 2t + t^3$

$a = 2 + t^2$

$R(2 + t^2, 0)$

$PR^2 = 4^2 + 4t^2 = 16^2$

$4t^2 = 12^2 \Rightarrow t = 3^2$

$a = 5$, $R(5, 0)$

Focus $(1, 0)$

$(1, 0)$ & $(5, 0)$ will be the end points of diameter

Eq of circle is

$(x - 1)(x - 5) + y^2 = 0$

$x^2 + y^2 - 6x + 5 = 0$

14. Let $X = R \times R$. Define a relation R on X as :

$(a_1, b_1) R (a_2, b_2) \Rightarrow b_1 = b_2$.

Statement-I : R is an equivalence relation.

Statement-II : For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y) R (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement-I and Statement-II are false.

(2) Statement-I is true but Statement-II is false.

(3) Both Statement-I and Statement-II are true.

(4) Statement-I is false but Statement-II is true.

Ans. (2)

Sol. Statement – I :

Reflexive : $(a_1, b_1) R (a_1, b_1) \Rightarrow b_1 = b_1$ True

Symmetric : $(a_1, b_1) R (a_2, b_2) \Rightarrow b_1 = b_2$

$(a_2, b_2) R (a_1, b_1) \Rightarrow b_2 = b_1$ True

Transitive : $(a_1, b_1) R (a_2, b_2) \Rightarrow b_1 = b_2$

& $(a_2, b_2) R (a_3, b_3) b_2 = b_3$

$\Rightarrow (a_1, b_1) R (a_3, b_3) \Rightarrow$ True $b_1 = b_3$

Hence Relation R is an equivence relation

Statement-I is true.

For statement – II $\Rightarrow y = b$ so False

১৫. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1,\frac{1}{2}\right)$, is:

(১) $\frac{2}{3}\sqrt{15}$ (২) $\frac{5}{3}\sqrt{15}$

(৩) $\frac{1}{3}\sqrt{15}$ (৪) $\sqrt{15}$

Ans. (১)

Sol. $T=S_1$

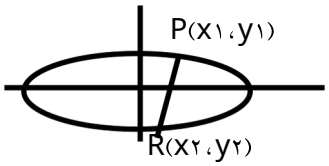
$\frac{x\cdot 1}{4}+\frac{y\cdot\frac{1}{2}}{2}=\frac{1}{4}+\frac{1}{8}$

$x+y=\frac{3}{2}$

solve with ellipse

$PR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{2}|x_2-x_1|$



$y_2=\frac{3}{2}-x_2$

$y_1=\frac{3}{2}-x_1$

$y_2-y_1=x_1-x_2$

$x^2+2y=4$

$x^2+2\left(\frac{3}{2}-x\right)^2=4$

$6x^2-12x+1=0$

$x_1+x_2=2$

$x_1x_2=1/6$

$|x_2-x_1|=\sqrt{(x_1+x_2)^2-4x_1x_2}$

$=\sqrt{4-4/6}$

$PR=\sqrt{2}.2.\frac{\sqrt{5}}{\sqrt{2}\sqrt{3}}=\frac{2}{3}\sqrt{15}$

১৬. Let $A=[a_{ij}]$ be a 3×3 matrix such that $\begin{bmatrix}0\\0\\0\end{bmatrix}$ $A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}4\\1\\3\end{bmatrix}$, $A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}2\\0\\0\end{bmatrix}$ and $A\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals:

(১) -1 (২) 0

(৩) 2 (৪) 1

Ans. (১)

Sol. Let $A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix}$

$A\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow a_{21}=0, a_{31}=0$

$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}2\\0\\0\end{bmatrix}\Rightarrow a_{11}+a_{12}=2$

$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}4\\1\\3\end{bmatrix}\Rightarrow a_{11}+a_{13}=4$

$2a_{11}+a_{12}+a_{13}=1$

$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}2\\0\\0\end{bmatrix}\Rightarrow 2a_{11}+a_{21}+2a_{12}=0\Rightarrow a_{12}=0$

$2a_{11}+a_{21}+2a_{22}=0\Rightarrow a_{22}=-1$

$2a_{11}+a_{21}+2a_{23}=0$

$-4a_{23}+3a_{23}=1\Rightarrow a_{23}=-1$

১৭. The number of complex numbers z, satisfying $|z|=1$ and $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$, is:

(১) ৬ (২) ৪

(৩) ১০ (৪) ৮

Ans. (৪)

Sol. $z=e^{i\theta}$

$\frac{z}{\bar{z}}=e^{i2\theta}$

$\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1\Rightarrow\left|e^{i2\theta}+e^{-i2\theta}\right|=1\Rightarrow|\cos 2\theta|=\frac{1}{2}$

8 solution in $(0,2\pi)$

১৮. If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z-3}{-3}$ and $\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then $m+n$ is equal to:

(১) ৬ (২) ৯

(৩) ২১ (৪) ১৪

Ans. (২)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = -\hat{i} + 3\hat{j} - 5\hat{k}$

$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix}$

$= -2\hat{i} - \hat{j}$

$\vec{b} - \vec{a} = -2\hat{i} + \hat{j} - 2\hat{k}$

$S_d = \frac{|(\vec{b} - \vec{a}) \cdot (\vec{p} \times \vec{q})|}{|\vec{p} \times \vec{q}|}$

$= \sqrt{5}$

$S_d^2 = \frac{4}{5}$

$m = 4, n = 5 \Rightarrow m + n = 9$

19. If $I = \int_0^{\frac{\pi}{2}} \frac{\sin^3 2x}{\sin^3 2x + \cos^3 2x} dx$,

then $\int_0^{2\pi} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$ equals:

(1) $\frac{\pi^2}{16}$ (2) $\frac{\pi^2}{4}$

(3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

Ans. (1)

Sol. For I

Apply king (P–5) and add

$2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$

$I_2 = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$

Apply king and add

$I_2 = \frac{\pi}{4} \int_0^{\pi/2} \frac{\tan x \sec^2 x}{\tan^4 x + 1} dx$

put $\tan x = t$

$= \frac{\pi}{8} \int_0^{\infty} \frac{dt}{t^2 + 1}$

$= \frac{\pi}{8} \cdot \frac{\pi}{2} = \frac{\pi^2}{16}$

20. $\lim_{x \to \infty} \frac{(2x^2 + 3x + 5)^{\frac{x}{2}}}{(3x^2 + 5x + 4)^{\left(\sqrt{3x^2 + 2}\right)^x}}$ is equal to:

(1) $\frac{2}{\sqrt{3e}}$ (2) $\frac{2e}{\sqrt{3}}$

(3) $\frac{2e}{3}$ (4) $\frac{2}{3\sqrt{e}}$

Ans. (4)

Sol. $\lim_{x \to \infty} \frac{\left(1 + \frac{3}{x} + \frac{5}{x^2}\right)^{\frac{x}{2}}}{\left(1 + \frac{5}{x} + \frac{4}{x^2}\right)^{\frac{x}{3x}}}$

$= \lim_{x \to \infty} \frac{e^{\frac{3x}{2} \cdot \frac{1}{x}}}{e^{\frac{5x}{3x}}}$

$= \frac{e^{\frac{1}{2}}}{e^{\frac{5}{3}}} = e^{\frac{1}{2} - \frac{5}{3}} = e^{-\frac{7}{6}}$

SECTION–B

21. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____.

Ans. (17280)

Sol. A : number of ways that all boys sit together $= 5! \times 5!$

B : number of ways if no 2 boys sit together $= 4! \times 5!$

$A \cap B = \phi$

Required no. of ways $= 5! \times 5! + 4! \times 5! = 17280$

22. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with $\text{Im}(\alpha) > \text{Im}(\beta)$. Let $P_n = \alpha^n - \beta^n$. If

$P_3 = 5\sqrt{7}i$, $P_4 = 3\sqrt{7}i$, $P_5 = 11\sqrt{7}i$ and

$P_6 = 45\sqrt{7}i$, then $|P_4 - P_1|$ is equal to _____.

Ans. (31)

Sol. $\alpha + \beta = a$ $\quad \alpha\beta = -b$

$P_n = aP_{n-1} + bP_{n-2}$

$45\sqrt{7}i = a(11\sqrt{7}i) + b(3\sqrt{7}i)$

$45 = 11a - 3b \quad \ldots(1)$

and

$P_٥ = aP_٤ + bP_٣$

$١١\sqrt{٧}i = a(٣\sqrt{٧}i) + b(٥\sqrt{٧}i)$

$١١ = -٣a - ٥b$ . . .(٢)

$a = ٣ ، b = -٤$

$|4\alpha + 4\beta| = \sqrt{(4\alpha + 4\beta)^٢ - 4\cdot 4\alpha\beta}$

$= \sqrt{٦٣ - ٤.٤٤}$

$= \sqrt{٣١٠٢٤} = \sqrt{٩٦١} = ٣١$

٢٣. The focus of the parabola $y^٢ = ٤x + ١٦$ is the centre of the circle C of radius ٥. If the values of λ, for which C passes through the point of intersection of the lines $٣x - y = ٠$ and $x + \lambda y = ٤$, are $\lambda_١$ and $\lambda_٢$, $\lambda_١ > \lambda_٢$, then $١٢\lambda_١ + ٢٩\lambda_٢$ is equal to ___________.

Ans. (١٥)

Sol. $y^٢ = ٤(x + ٤)$

Equation of circle

$(x + ٣)^٢ + y^٢ = ٢٥$

Passes through the point of intersection of two lines $٣x - y = ٠$ and $x + \lambda y = ٤$ which is

$\left(\frac{٤}{١ + ٣\lambda}, \frac{١٢}{١ + ٣\lambda}\right)$, after solving with circle, we get

$\lambda = -\frac{٧}{٦}, ١$

$12\lambda_١ + ٢٩\lambda_٢$

$-١٤ + ٢٩ = ١٥$

٢٤. The variance of the numbers ٨, ٢١, ٣٤, ٤٧, ..., ٣٢٠, is ___________.

Ans. (٨٧٨٨)

Sol. Var(٨, ٢١, ٣٤, ٤٧, ..., ٣٢٠)

Var(٠, ١٣, ٢٦, ٣٩, ..., ٣١٢)

$١٣^٢$Var(٠, ١, ٢, ..., ٢٤)

$١٣^٢$Var(١, ٢, ٣, ..., ٢٥)

So, $\sigma^2 = 132\left(\frac{٢٥^٢ - ١}{١٢}\right) = ٨٧٨٨$

Alternate solution

$٨ + (n - ١)١٣ = ٣٢٠$

$١٣n = ٣٢٥$

$n = ٢٥$

no. of terms = ٢٥

$\text{mean} = \frac{\sum x_i}{n} = \frac{8 + ٢١ + . . + ٣٢٠}{٢٥} = \frac{\frac{٢٥}{٢}(٨ + ٣٢٠)}{٢٥}$

$\text{variance} = \frac{\sum x_i^٢}{n} - (\text{mean})^٢$

$= \frac{٨^٢ + ٢١^٢ + . . . + ٣٢٠^٢}{١٣} - (164)^٢$

$= ٨٧٨٨$

٢٥. The roots of the quadratic equation $٣x^٢ - px + q = ٠$ are ١٠^{th} and ١١^{th} terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first ١١ terms of this arithmetic progression is ٨٨, then $q - ٢q$ is equal to ___________.

Ans. (٤٧٤)

Sol. $S_{١١} = \frac{١١}{٢}(٢a + ١٠d) = ٨٨$

$a + ٥d = ٨$

$a = 8 - 5\left(\frac{٣}{٢}\right) = \frac{١}{٢}$

Roots are

$T_{١٠} = a + 9d = \frac{١}{٢} + 9\left(\frac{٣}{٢}\right) = 14$

$T_{١١} = a + 10d = \frac{١}{٢} + 10\left(\frac{٣}{٢}\right) = \frac{٣١}{٢}$

$\frac{p}{٣} = T_{١٠} + T_{١١} = ١٤ + \frac{٣١}{٢} = \frac{٥٩}{٢}$

$p = \frac{١٧٧}{٢}$

$\frac{q}{٣} = T_{١٠} \cdot T_{١١} = ٧ \times ٣١ = ٢١٧$

$q = ٦٥١$

$q - ٢p$

$= ٦٥١ - ١٧٧$

$= ٤٧٤$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY23 nd JANUARY 2025) TIME : 3 : 00PM TO 6 : 00 PM

PHYSICS

SECTION-A

٢٦. A ball having kinetic energyKE is projected at an angle of ٦٠° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight?

(١) $\frac{(KE)}{٨}$ (٢) $\frac{(KE)}{٤}$

(٣) $\frac{(KE)}{١٦}$ (٤) $\frac{(KE)}{٢}$

Ans. (٢)

Sol. Initial K.E.

$K.E. = \frac{١}{٢} mu$

Speed at heighest point

$V = u\cos ٦٠° = \frac{u}{٢}$

$KE = \frac{١}{٢} m\left(\frac{u}{٢}\right)^{٢}$

$= \frac{١}{٤}\left(\frac{١}{٢} mu^{٢}\right)$

$KE = \frac{E}{٤}$

٢٧. Two charges ٧ μC and − ٤ μC are placed at (− ٧ cm, ٠, ٠) and (٧ cm, ٠, ٠) respectively. Given $\frac{١}{٤\pi\varepsilon_{٠}} = ٨.٨٥ \times ١٠^{-١٢}$ C$^{-٢}$ N$^{-١}$ m^{٢}, the electrostatic potential energy of the charge configuration is :

(١) − ١.٥ J (٢) − ٢.٠ J

(٣) − ١.٢ J (٤) − ١.٨ J

Ans. (٤)

Sol. P.E. of two charges

$u = \frac{١}{4\pi\varepsilon_{٠}} \frac{q_{١} q_{٢}}{r}$

$r = \sqrt{(x_{٢} - x_{١})^{٢} + (y_{٢} - y_{١})^{٢} + (z_{٢} - z_{١})^{٢}}$

$= ١٤$ cm

$u = \frac{٩ \times ١٠^{٩} \times ٧ \times ١٠^{-٦} \times (-٤) \times ١٠^{-٦}}{١٤ \times ١٠^{-٢}}$

$= -١.٨$ J

TEST PAPER WITH SOLUTION

٢٨. The refractive index of the material ofa glass prism is $\sqrt{٣}$. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism?

(١) ٥٠° (٢) ٦٠°

(٣) ٥٨° (٤) ٤٨°

Ans. (٢)

Sol. $\mu = \frac{\sin\left(\frac{A + \delta_{min}}{٢}\right)}{\sin\frac{A}{٢}}$

Given $\delta_{min} = A$

$\sqrt{٣} = \frac{\sin A}{\sin\frac{A}{٢}} = \frac{٢\sin\frac{A}{٢}\cos\frac{A}{٢}}{\sin\frac{A}{٢}}$

$\cos\frac{A}{٢} = \frac{\sqrt{٣}}{٢}$

$A = ٦٠°$

٢٩. The equation of a transverse wave travelling along a string is $y(x, t) = ٤.٠ \sin(٢٠ \times ٣٠\, x + ٦٠٠\, t)$ mm, where x is in the mm and t is in second. The velocity of the wave is :

(١) + ٣٠ m/s (٢) − ٦٠ m/s

(٣) − ٣٠ m/s (٤) + ٦٠ m/s

Ans. (٣)

Sol. $y = ٤\sin(٢٠ \times ١٠\, x + ٦٠٠\, t)$

Here $\omega = ٦٠٠$ s$^{-١}$

$k = ٢٠ \times ١٠$ mm$^{-١}$

$v = \frac{\omega}{k} = \frac{٦٠٠}{٢٠ \times ١٠^{٣}}$

$= ٣٠ \times ١٠^{٣}$ mm/s

$= ٣٠$ m/s

& direction is towards –ve x axis

$v = -٣٠$ m/s

٣٠. The energy of a system is given as $E(t) = \alpha e^{\beta^2 - \gamma t}$ where t is the time and $\gamma = ٠.٣$ s. The errors in the measurement of γ and t are ١.٢٪ and ١.٦٪, respectively. At t = ٥ s, maximum percentage error in the energy is :

(١) ٤٪ (٢) ١١.٦٪

(٣) ٦٪ (٤) ٨.٤٪

Ans. (٣)

Sol. $E = \alpha e^{-\gamma t}$

$\ln E = ٣ \ln \alpha - \gamma t$

$\left(\frac{dE}{E}\right)_{max} = \frac{٣d\alpha}{\alpha} + \frac{dt}{t} + \gamma t$

$= ٣ \times ١.٢٪ + (٠.٣ \times ١.٦ \times ٥)٪$

$= ٦٪$

٣١. In photoelectric effect an em-wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is ٢.١٤ eV and stopping potential is ٢V, what is the wavelength of the em-wave?
(Given hc = ١٢٤٢ eVnm where h is the Planck's constant and c is the speed of light in vaccum.)

(١) ٤٠٠ nm (٢) ٦٠٠ nm

(٣) ٢٠٠ nm (٤) ٣٠٠ nm

Ans. (٤)

Sol. $eV = E - \phi$

$٢ eV = E - ٢.١٤ eV$

$E = ٤.١٤ eV$

$E = \frac{hc}{\lambda}$

$\lambda = \frac{١٢٤٢}{٤.١٤} = ٣٠٠ nm$

٣٢. A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\theta(t) = ٥t - ٨t$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t.
How much power is delivered by the applied torque, when t = ٢s?

(١) ٦٠ MR^2 (٢) ٧٢ MR^2

(٣) ١٠٨ MR^2 (٤) ٨ MR^2

Ans. (١)

Sol. $\theta = ٥t - ٨t$

$\omega = \frac{d\theta}{dt} = ١٠t - ٨$

$\alpha = \frac{d\omega}{dt} = ١٠$

$\tau_p = \tau\omega$

$= (I\alpha)\omega$

$= \left(\frac{١}{٢}mR^٢\right)(١٠)(١٠t - ٨)$

Put t = ٢

$p = ٦٠ mR^2$

٣٣. Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is $P_١$. The reading of the pressure gauge falls to $P_٢$ when the valve is opened. The speed of water flowing in the pipe is proportional to

(١) $\sqrt{P_١ - P_٢}$ (٢) $(P_١ - P_٢)^٢$

(٣) $(P_١ - P_٢)^٤$ (٤) $P_١ - P_٢$

Ans. (١)

Sol. By Bernoulli equation

$P_١ + \frac{١}{٢} \times \rho \times ٠^٢ = P_٢ + \frac{١}{٢}\rho v^2$

$v = \sqrt{٢(P_١ - P_٢)}$

٣٤. Match List-I with List-II.

List-I	List-II
(A) Permeability of free space	(I) $M^١L^١T^{-٢}A^{-٢}$
(B) Magnetic field	(II) $MT^{-٢}A^{-١}$
(C) Magnetic moment	(III) $MLT^{-٢}A^{-١}$
(D) Torsional constant	(IV) $L^٢A^١$

Choose the correct answer from the options given below : (١) (A)-(I), (B)-(IV), (C)-(II), (D)-(III) (٢) (A)-(II), (B)-(I), (C)-(III), (D)-(IV) (٣) (A)-(IV), (B)-(III), (C)-(I), (D)-(II) (٤) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Ans. (٤)

Sol. $B = \frac{\mu_0 I}{2r}$

$[\mu_0] = \left[\frac{Br}{I}\right] = \left[\frac{MLT^{-2}A^{-1}L}{A}\right] = [MLTA^{-2}]$

magnetic field $F = qvB$

$B = \left[\frac{MLT^{-2}}{AT\,L/T}\right] = [MT^{-2}A^{-1}]$

$[M] = [NTA^{-2}] = [M][L] = [ML]$

$[\mu_0] = c[\varepsilon_0]\ c = \frac{[L]}{[\varepsilon_0][T]} = [LT^{-1}]$

35. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the moon is neglected.

(1) 1 day
(2) 81 days
(3) 27 days
(4) 3 days

Ans. (1)

Sol. $T^2 \propto R^3$

$\left(\frac{T_m}{T_s}\right)^2 = \frac{R^3}{(R/9)^3}$

$\frac{T_m}{T_s} = (3)^3$

$T_s = \frac{27}{27} = 1$ day

36. Two point charges $-4\ \mu C$ and $4\ \mu C$, constituting an electric dipole, are placed at $(-9, 0, 0)$ cm and $(9, 0, 0)$ cm in a uniform electric field of strength 10^4 NC^{-1}. The work done on the dipole in rotating it from the equilibrium through $180°$ is :

(1) 14.4 mJ
(2) 18.4 mJ
(3) 12.4 mJ
(4) 16.4 mJ

Ans. (1)

Sol. $U = -PE\cos\theta$

$w_{ext} = \Delta U = U - U_f = -PE\cos 180° + PE\cos 0°$

$w_{ext} = 2PE$

$= 2 \times (4 \times 10^{-6})(18) \times 10^4$

$= 144 \times 10^{-2}$

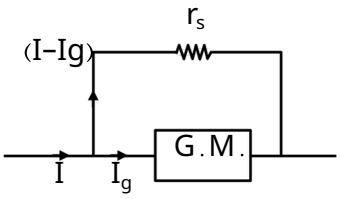
$= 14.4$ mJ

37. A galvanometer having a coil of resistance 30 Ω need 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added to the galvanometer should be $\frac{30}{X}\ \Omega$, where X is

(1) 447
(2) 298
(3) 149
(4) 596

Ans. (3)

Sol.



$I_g R_g = (I - I_g) r_s$

$20 \times 10^{-3} \times 30 = (3 - 0.02) \times r_s$

$r_s = \frac{0.6}{2.98} \times 30 = x$

$x = \frac{2.98 \times 30}{0.6} = 149$

38. The width of one of the two slits in Young's double slit experiment is d while that of the other slit is xd. If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is 9 : 4 then what is the value of x?
(Assume that the field strength varies according to the slit width.)

(1) 2
(2) 3
(3) 5
(4) 4

Ans. (3)

Sol. $I \propto (\text{width})^2$

$\left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{9}{4}$

$\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{3}{2}$

$\frac{(x+1)}{(x-1)} = \frac{3}{2}$

$2x - 2 = 2x + 2$

$x = 5$

३९. Given below are two statements. One is labelled as Assertion (A) and the other is labelled as Reason (R). Assertion (A) : The binding energy per nucleon is found to be practically independent of the atomic number A, for nuclei with mass numbers between ३० and १७०. Reason (R) : Nuclear force is long range. In the light of the above statements, choose the correct answer from the options given below : (१)(A) is false but (R) is true (२)(A) is true but (R) is false (३)Both (A) and (R) are true and (R) is the correct explanation of (A) (४)Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

Ans. (२)

Sol. Conceptual

४०. Water of mass m gram is slowly heated to increase the temperature from T to T. The change in entropy of the water, given specific heat of water is १ Jkg K, is :

(१)zero (२)$m(T-T_{२})_{१}$

(३) $m\ln\frac{T}{T_{२}}$ (४) $m\ln\frac{T}{T_{१}}$

Ans. (४)

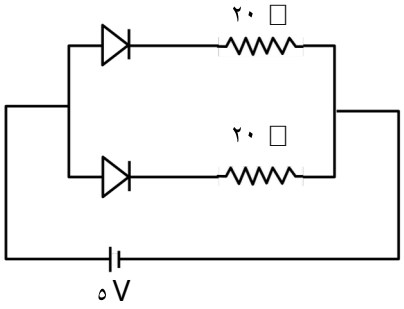
Sol. $dQ = msdT$

$dS = \frac{dQ}{T} = \frac{msdT}{T}$

$\Delta S = \int\frac{msdT}{T} = ms\ln\frac{T_f}{T_i}$

$\Delta S = m\ln\frac{T_{२}}{T_{१}}$

४१. What is the current through the battery in the circuit shown below?



(१)१.० A (२)१.५ A

(३)०.५ A (४)०.२५ A

Ans. (३)

Sol. Both are forward biased

hence $R_{eq} = १०\ \Omega$

$i = \frac{V}{R} = \frac{५}{१०} = \frac{१}{२}$ A

४२. A plane electromagnetic wave of frequency २० MHz travels in free space along the +x direction. At a particular point in space and time, the electric field vector of the wave is $E_y = ९.३\ Vm^{-१}$. Then, the magnetic field vector of the wave at that point is–

(१)$B_z = ९.३ \times १०^{-८}\ T$ (२)$B_z = १.५५ \times १०^{-८}\ T$

(३)$B_z = ६.२ \times १०^{-८}\ T$ (४)$B_z = ३.१ \times १०^{-८}\ T$

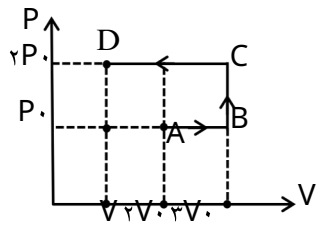
Ans. (४)

Sol. $E = BC$

$९.३ = B \times ३ \times १०^{८}$

$B = \frac{९.३}{३ \times १०^{८}} = ३.१ \times १०^{-८}\ T$

४३.



Using the given P–V diagram, the work done by an ideal gas along the path ABCD is– (२)३ PV

(१)४ PV (३)–४ PV (४)–३ PV

Ans.(४)

Sol. $w_{ABCD} = w_{AB} + w_{BC} + w_{CD}$

$= PV + 0 + (-2P \times 2V)$

$= PV - 4PV$

$= -3PV$

44. A concave mirror of focal length f in air is dipped in a liquid of refractive index μ. Its focal length in the liquid will be :

(1) $\frac{f}{\mu}$ (2) $\frac{f}{(\mu - 1)}$

(3) μf (4) f

Ans. (4)

Sol. Focal length of mirror will not change because focal length of mirror doesn't depend on medium.

45. A massless spring gets elongated by amount x_1 under a tension of 5N. Its elongation is x_2 under the tension of 7N. For the elongation of $(5x_1 - 2x_2)$, the tension in the spring will be :

(1) 15 N (2) 20 N

(3) 11 N (4) 39 N

Ans. (3)

Sol. $kx_1 = 5N$

$kx_2 = 7N$

$k(5x_1 - 2x_2) = 5kx_1 - 2kx_2$

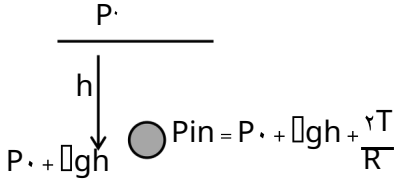
$= 5 \times 5 - 2 \times 7 = 11$ N

SECTION-B

46. An air bubble of radius 1.0 mm is observed at a depth of 20 cm below the free surface of a liquid having surface tension 0.095 J/m^2 and density 10^3 kg/m^3. The difference between pressure inside the bubble and atmospheric pressure _____ N/m^2. (Take g = 10 m/s^2)

Ans. (2190)

Sol.



$\Delta P = P_{in} - P$

$= \rho gh + \frac{2T}{R} = \frac{1000 \times 10 \times 20}{100} + \frac{2 \times 0.095}{10^{-3}}$

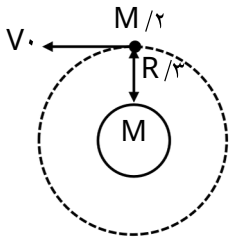
$= 2000 + 190$

$= 2190$

47. A satellite of mass $\frac{M}{2}$ is revolving around earth in a circular orbit at a height of $\frac{R}{3}$ from earth surface. The angular momentum of the satellite is $M\sqrt{\frac{GMR}{x}}$. The value of x is _____, where M and R are the mass and radius of earth, respectively (G is the gravitational constant)

Ans. (3)

Sol. (i) If earth is assumed to be stationary



orbital velocity $v = \sqrt{\frac{GM}{4R/3}} = \sqrt{\frac{3GM}{4R}}$

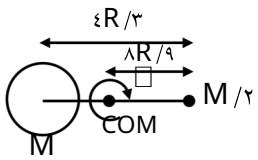
Angular momentum of satellite $= v \cdot \frac{4R}{3}$

$= \frac{M}{2} \cdot \sqrt{\frac{3GM}{4R}} \cdot \frac{4R}{3}$

$= M\sqrt{\frac{GMR}{3}}$

$x = 3$

(ii) Since mass of satellite is comparable to the mass of earth.



$\frac{G.M.M}{(4R)^2} = \frac{M}{2} \cdot \frac{2 \cdot 8R}{9}$

$\sqrt{\frac{81GM}{128R^3}}$

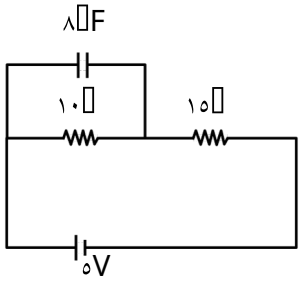
Angular momentum of satellite about common centre of mass,

$L = \frac{M}{2} \cdot \left(\frac{8R}{9}\right) \cdot \left(\frac{4}{3}\right)$

$L = M\sqrt{\frac{GMR}{81}}$

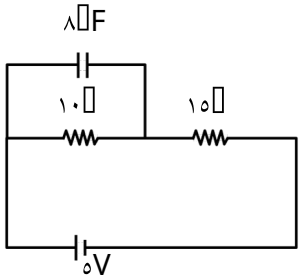
$x = \frac{81}{8} \simeq 10$

٤٨. At steady state the charge on the capacitor, as shown in the circuit below, is _____ μC.



Ans. (١٦)

Sol.



$i = \frac{5}{25}$

$Q = CV$

$Q = (8 \times 10^{-6}) \left(\frac{5}{25}\right) 10$

$Q = \frac{8 \times 5 \times 10^{-2}}{25} = 16\ \mu C$

٤٩. A time varying potential difference is applied between the plates of a parallel plate capacitor of capacitance ٢.٥ μF. The dielectric constant of the medium between the capacitor plates is ١. It produces an instantaneous displacement current of ٠.٢٥ mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be _____ Vs^{-1}

Ans. (١٠٠)

Sol. $\frac{CdV}{dt} = I_d$

$\frac{dV}{dt} = \frac{I_d}{\epsilon}$

$= \frac{0.25 \times 10^{-3}}{2.5 \times 10^{-6}}$

$= 100$

٥٠. In a series LCR circuit, a resistor of ٣٠٠ Ω, a capacitor of ٢٥ nF and an inductor of ١٠٠ mH are used. For maximum current in the circuit, the angular frequency of the ac source is _____ $\times 10^{4}$ radians^{-1}s.

Ans. (٢)

Sol. $= \frac{1}{\sqrt{LC}}$

$= \frac{1}{\sqrt{25 \times 10^{-9} \times 100 \times 10^{-3}}}$

$= \frac{10^{6}}{5 \times 10} \times 2$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025) TIME : 3 : 00 PM TO 6 : 00 PM

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

٥١. The effect of temperature reactions on spontaneity of are represented as:

	□H	□S	Temperature	Spontaneity
(A)	+	–	any T	Non spontaneous
(B)	+	+	low T	spontaneous
(C)	–	–	low T	Non spontaneous
(D)	–	+	any T	spontaneous

(١) (B) and (D) only
(٢) (A) and (D) only
(٣) (B) and (C) only
(D) (A) and (C) only

Ans. (٣)

Sol. ∵ □G = □H – T□S
For spontaneity of reaction : □G = –ve

٥٢. Standard electrode potentials for a few half cells are mentioned below:

$E^{o}_{Cu^{2□}/Cu}$ □0.34V, $E^{o}_{Zn^{2□}/Zn}$ □ □0.76V

$E^{o}_{Ag^{□}/Ag}$ □٠.٨٠V, $E^{o}_{Mg^{2□}/Mg}$ □ □٢.٣٧V

Which one of the following cells gives the most negative value of □G°?

(١) Zn|Zn^{2+}(١M)||Ag^{+}(١M)|Ag
(٢) Zn|Zn^{2+}(١M)||Mg^{2+}(١M)|Mg
(٣) Ag|Ag^{+}(١M)||Mg^{2+}(١M)|Mg
(٤) Cu|Cu^{2+}(١M)||Ag^{+}(١M)|Ag

Ans. (١)

Sol. ∵ □G° = –nFE°

Option (١) E° = ٠.٨ + ٠.٧٦
= ١.٥٦ V
□□G° = –٢ × F × ١.٥٦
= –٣.١٢ V

Option (٢) E° = –٢.٣٧ + ٠.٧٦
= –١.٦١ V
□□G° = –٢ × F × (–١.٦١)
= +٣.٢٢ V

Option (٣) E° = –٢.٣٧ – ٠.٨
= –٣.١٧ V
□□G° = –٢ × F × (–٣.١٧)
= +٦.٣٤

Option (٤) E° = ٠.٨ – ٠.٣٤
= ٠.٤٦ V
□G° = –٢ × F × ٠.٤٦
= –٠.٩٢ V

٥٣. The □□– Helix and □□– Pleated sheet structures of protein are associated with its:

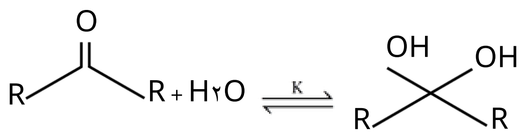
(١) quaternary structure
(٢) primary structure
(٣) secondary structure
(٤) tertiary structure

Ans. (٣)

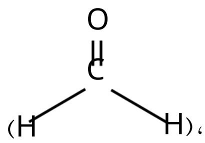
Sol. □-helix and □-pleated sheet belongs to secondary structure of protein, which have hydrogen bonds.

٥٤. Given below are two statements:

Consider the following reaction

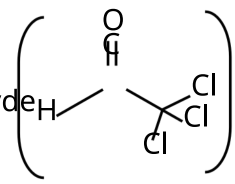


Statement (I) : In the case of formaldehyde



, K is about ٢٢٨٠, due to small substituents, hydration is faster.

Statement (II) : In the case of trichloro acetaldehyde



, K is about ٢٠٠٠ due to –I effect of –Cl.

In the light of the above statements, choose the correct answer from the options given below:

(١) Statement I true but Statement II is false
(٢) Both Statement I and Statement II are true
(٣) Statement I is false but Statement II is true
(٤) Both Statement I and Statement II are false

Ans. (٢)

Sol. k_{eq} ٢٢٨٠ is for HCHO
keq = ٢٠٠٠ is for chloral
Both data is given in clayden and warren book.
keq < ١ because HCHO and chloral are more electrophilic.

٥٥. Consider the reaction

$X_2Y(g) \rightleftharpoons X_2(g) + \frac{1}{2}Y_2(g)$

The equation representing correct relationship between the degree of dissociation (x) of $X_2Y(g)$ with its equilibrium constant Kp is _____.

Assume x to be very very small.

(١) $x = \sqrt[3]{\frac{2Kp}{p}}$ (٢) $x = \sqrt[3]{\frac{2Kp^2}{p}}$

(٣) $x = \sqrt[3]{\frac{Kp}{2p}}$ (٤) $x = \sqrt[2]{\frac{Kp}{p}}$

Ans. (٢)

Sol. ١ mole

$X_2Y(g) \rightleftharpoons X_2(g) + \frac{1}{2}Y_{2(g)}$

١-x mole x mole $\frac{x}{2}$ mole

$P_{X_2Y} = \frac{1-x}{1-\frac{x}{2}}P$

$P_{X_2} = \frac{x}{1-\frac{x}{2}}P$

$P_{Y_2} = \frac{x/2}{1-\frac{x}{2}}P$

$K_p = \frac{x}{1-\frac{x}{2}}P \cdot \left(\frac{x}{2+\frac{x}{2}}P\right)^{\frac{1}{2}} \Big/ \frac{1-x}{1-\frac{x}{2}}P$

$Kp = \frac{x}{1-x}\left(\frac{x}{2+\frac{x}{2}}\right)^{\frac{1}{2}}p$

$\because$ x to be very very small

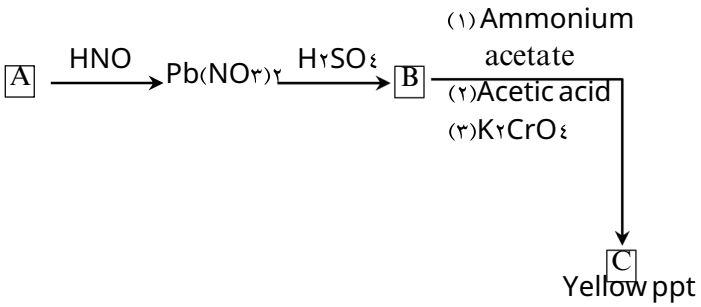
$K_p = \frac{x^{3/2}}{(2)^{\frac{1}{2}}}P^{-\frac{1}{2}}$

$x^{\frac{3}{2}} = \frac{K_p 2^{\frac{1}{2}}}{P^{-\frac{1}{2}}}$

$x^3 = \frac{K_p^2 2}{P}$

$x = \left(\frac{K_p^2 2}{P}\right)^{\frac{1}{3}}$

٥٦. Identify A, B and C in the given below reaction seque



(١) $PbCl_2$, $PbSO_4$, $PbCrO_4$

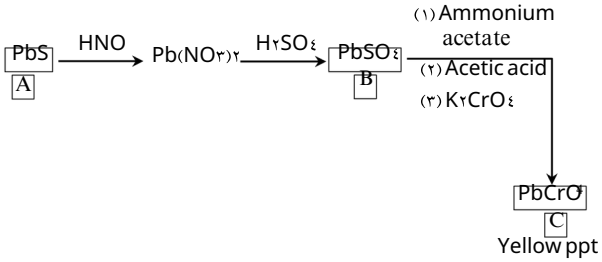
(٢) PbS, $PbSO_4$, $PbCrO_4$

(٣) PbS, $PbSO_4$, $Pb(CH_3COO)_2$

(٤) $PbCl_2$, $Pb(SO_4)_2$, $PbCrO_4$

Ans. (٢)

Sol.



٥٧. Given below are two statements: Statement (I): The boiling points of alcohols and phenols increase with increase in the number of C-atoms. Statement (II): The boiling points of alcohols and phenols are higher in comparison to other class of compounds such as ethers, haloalkanes.

In the light of the above statements, choose the correct answer from the options given below:

(١) Both Statement I and Statement II are false

(٢) Statement I is false but Statement II is true

(٣) Statement I is true but Statement II is false

(٤) Both Statement I and Statement II are true

Ans. (٤)

Sol. B.P. $\propto$ M.W.

B.P. $\propto$ Inter molecular hydrogen bonding

Alcohol & Phenol have intermolecular H-bonding

৫৮. When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by ১০ mm of Hg. The mole fraction of the solute in the solution is ০.২. What would be the mole fraction of the solvent if decrease in vapour pressure is ২০ mm of Hg ? (১) ০.৬ (৩) ০.২ (২) ০.৪ (৪) ০.৮

Ans. (১)

Sol. $\therefore P^\circ - P = X_{solute}$

and $\therefore 10 = 0.2$

$20 = 0.4$

$X_{solvent} = 1 - X_{solute}$

$= 1 - 0.4$

$= 0.6$

৫৯. Given below are two statements :

Statement (I) : For a given shell, the total number of allowed orbitals is given by n.

Statement (II) : For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.

In the light of the above statements, choose the correct answer from the options given below :

(১) Statement I is true but Statement II is false

(২) Statement I is false but Statement II is true

(৩) Both Statement I and Statement II are true

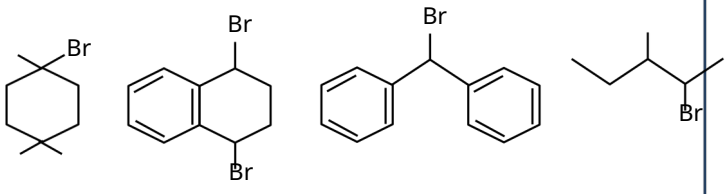
(৪) Both Statement I and Statement II are false

Ans. (৩)

Sol. For a shell total number of orbitals $= n^2$

Magnetic quantum number have values ($-l$ to $+l$) including ০.

৬০. The ascending order of relative rate of solvolysis of following compounds is



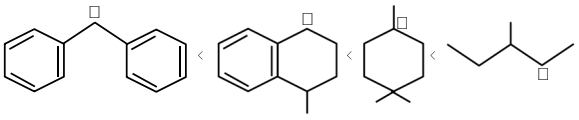
(A) (B) (C) (D)

(১) (D) > (A) > (B) > (C) (২) (C) > (B) > (A) > (D) (৩) (D) > (B) > (A) > (C) (৪) (C) > (D) > (B) > (A)

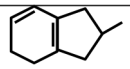
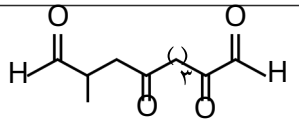
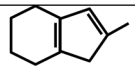
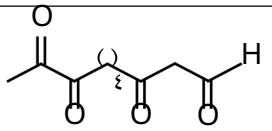
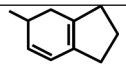
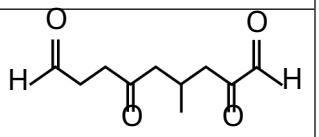
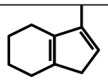
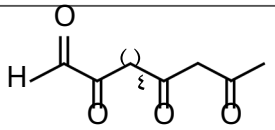
Ans. (১)

Sol. Solvolysis or S_N1 stability of carboccation

Stability order



৬১. Match List - I with List - II.

List - I (Isomers of $C_{10}H_{14}$)		List - II (Ozonolysis product)	
(A)		(I)	
(B)		(II)	
(C)		(III)	
(D)		(IV)	

Choose the correct answer from the options given below :

(১) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

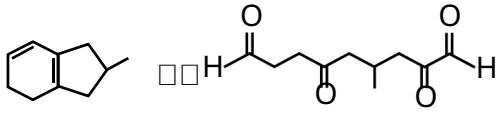
(২) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

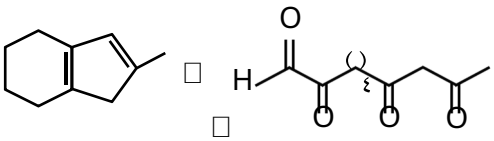
(৩) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

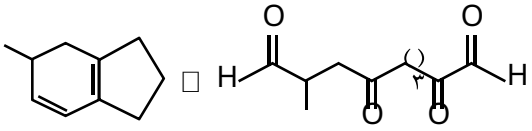
(৪) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

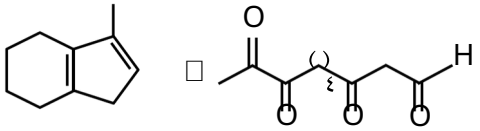
Ans. (২)

Sol. Ozonolysis product

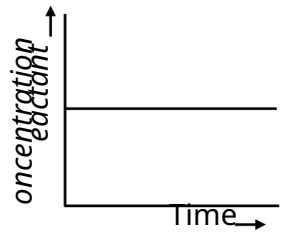
(A) 

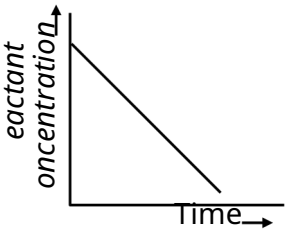
(B) 

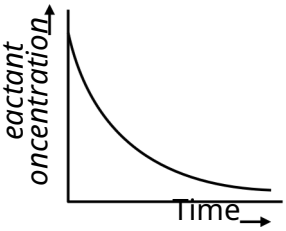
(C) 

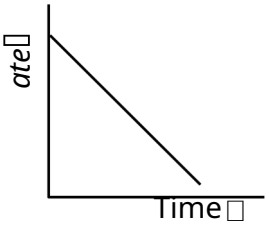
(D) 

٦٢. Which of the following graphs most appropriately represents a zero order reaction ?

(١) 

(٢) 

(٣) 

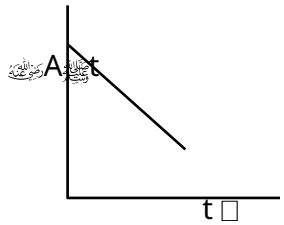
(٤) 

Ans. (٢)

Sol. For zero order reaction : A → P

Rate = k

$[A]_t = a_0 - kt$



٦٣. Match List - I with List - II.

List - I		List - II	
(A)	Bronze	(I)	Cu، Ni Fe،
(B)	Brass	(II)	Cr، Ni، C
(C)	UK silver coin	(III)	Cu، Zn Cu،
(D)	Stainless Steel	(IV)	Sn

Choose the correct answer from the options given below : (١)(A)-(IV)، (B)-(II)، (C)-(III)، (D)-(I) (٢)(A)-(IV)، (B)-(III)، (C)-(I)، (D)-(II) (٣)(A)-(III)، (B)-(I)، (C)-(IV)، (D)-(II) (٤)(A)-(III)، (B)-(IV)، (C)-(II)، (D)-(I)

Ans. (٢)

Sol. Bronze → Cu، Sn

Brass → Cu، Zn

UK silver coin → Cu، Ni

Stainless steel → Fe، Cr، Ni، C

٦٤. Identify the coordination complexes in which the central metal ion has t_{2g}^6 d configuration.

(A) $[FeO_4]^{2-}$

(B) $[Mn(CN)_6]^{3-}$

(C) $[Fe(CN)_6]^{4-}$

(D) $Cr_2(O-\overset{O}{\overset{\|}{C}}-Me)_4(H_2O)_2$

(E) $[NiF_6]^{4-}$

Choose the correct answer from the options given below :

(١) (C) and (E) only (٢) (B)، (C) and (D) only

(٣) (B) and (D) only (٤) (A)، (B) and (E) only

Ans. (٣)

Sol. $Fe^{+3} = [Ar]^{18} 3d^5$

$Mn^{+3} = [Ar]^{18} 3d^4$

$Fe^{+3} = [Ar]^{18} 3d^5$

$Cr^{+2} = [Ar]^{18} 3d^4$

$Ni^{+4} = [Ar]^{18} 3d^6$

٦٥. Given below are the atomic numbers of some group ١٤ elements. The atomic number of the element with lowest melting point is :

(١) ١٤ (٢) ٦

(٣) ٨٢ (٤) ٥٠

Ans. (٤)

Sol. Order of M.P. of group ١٤ : C > Si > Ge > Pb > Sn

element	M.P. (°C)
Z = ٦ = C	٣٧٣٠
Z = ١٤ = Si	١٤١٠
Z = ٣٢ = Ge	٩٣٧
Z = ٥٠ = Sn	٢٣٢
Z = ٨٢ = Pb	٣٢٧

٦٦. pH of water is ٧ at ٢٥°C. If water is heated to ٨٠°C، it's pH will :

(١) Decrease

(٢) Remains the same

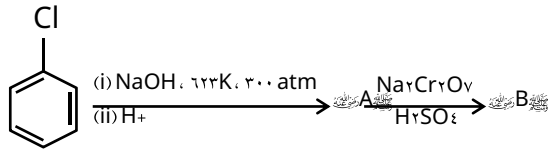
(٣) H^+ concentration increases، OH^- concentration decreases

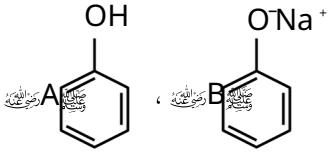
(٤) Increase

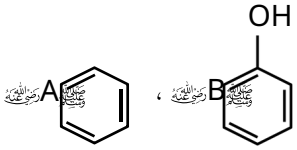
Ans. (١)

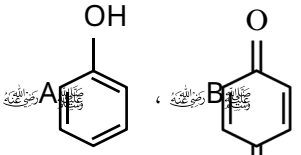
Sol. With increase in temperature، Kw of water increases So، degree of dissociation of water increase ∴ pH as well as pOH of water decrease.

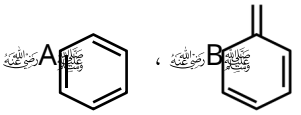
٦٧. Identify the products A and B, respectively in the following reaction :



(١) 

(٢) 

(٣) 

(٤) 

Ans. (٣)

Sol. A is phenol and B is para benzoquinone.

٦٨. Consider a binary solution of two volatile liquid components ١ and ٢ x_1 and y_1 are the mole fractions of component ١ in liquid and vapour phase, respectively. The slope and intercept of the linear plot of $\frac{1}{x_1}$ vs $\frac{1}{y_1}$ are given respectively as :

(١) $\frac{P_2^\circ}{P_1^\circ}, \frac{P_2^\circ - P_1^\circ}{P_1^\circ}$ (٢) $\frac{P_1^\circ}{P_2^\circ}, \frac{P_1^\circ - P_2^\circ}{P_2^\circ}$

(٣) $\frac{P_1^\circ}{P_2^\circ}, \frac{P_1^\circ - P_2^\circ}{P_2^\circ}$ (٤) $\frac{P_2^\circ}{P_1^\circ}, \frac{P_1^\circ - P_2^\circ}{P_2^\circ}$

Ans. (١)

Sol. $\because$ For liquid solution of two liquids '١' and '٢'

$P_1 = PTy_1 = Px_1$

$$= \frac{P_T}{x_1} = \frac{P_1^\circ}{y_1}$$

$$= \frac{x_1(P_1^\circ - P_2^\circ)}{x_1} = \frac{P_1^\circ}{y_1}$$

$$= \frac{P_2^\circ}{x_1} = (P_1^\circ - P_2^\circ)\frac{P_1^\circ}{y_1}$$

$$= \frac{1}{x_1} = \frac{P_1^\circ}{P_2^\circ} \cdot \frac{1}{y_1} - \frac{P_1^\circ - P_2^\circ}{P_2^\circ}$$

$$\text{Slope} = \frac{P_1^\circ}{P_2^\circ}$$

$$\text{Intercept} = \frac{P_1^\circ - P_2^\circ}{P_2^\circ}$$

٦٩. Given below are two statements about X-ray spectra of elements :

Statement (I) : A plot of $\sqrt{\nu}$ (ν = frequency of X-rays emitted) vs atomic mass is a straight line.

Statement (II) : A plot of $\sqrt{\nu}$ (ν = frequency of X-rays emitted) vs atomic number is a straight line.

In the light of the above statements choose the correct answer from the options given below :

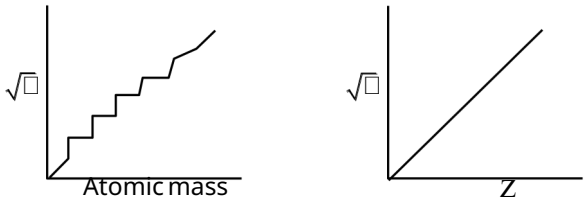
(١) Statement I is true but Statement II is false

(٢) Both Statement I and Statement II are true

(٣) Both Statement I and Statement II are false

(٤) Statement I is false but Statement II is true

Ans. (٣)

Sol. 

٧٠. Consider the following reactions

$$K_2Cr_2O_7 \xrightarrow[H_2O]{KOH} [A] \xrightarrow[H_2O]{H_2SO_4} B + K_2SO_4$$

The products A and B, respectively are :

(١) $K_2Cr(OH)_6$ and Cr_2O_3

(٢) K_2CrO_4 and Cr_2O_3

(٣) K_2CrO_4 and $K_2Cr_2O_7$

(٤) K_2CrO_4 and CrO

Ans. (٣)

Sol. $K_2Cr_2O_7 \xrightarrow[H_2O]{KOH} \underset{A}{K_2CrO_4} \xrightarrow[H_2O]{H_2SO_4} \underset{B}{K_2CrO_7} + K_2SO_4$

٧٥. A compound 'X' absorbs ٢ moles of hydrogen and 'X' upon oxidation with $KMnO_4/H^+$ gives

$CH_3-\overset{\overset{O}{\|}}{C}-CH_3$, $CH_3-\overset{\overset{O}{\|}}{C}-OH$ and $CH_3-\overset{\overset{O}{\|}}{C}-CH_2CH_2-\overset{\overset{O}{\|}}{C}-OH$

The total number of π bonds present in the compound 'X' is _____.

Ans. (٢٧)

Sol.

$$CH_3-\overset{\overset{CH_2}{\|}}{C}-CH_2-CH_2-CH=CH-CH_3 \xrightarrow[H^+]{KmnO_4}$$

CH_3 CH_3

OR

$$CH_3-\overset{\overset{CH}{\|}}{C}-CH_2-CH_2-CH=\overset{\overset{CH_3}{|}}{C}-CH_3$$

CH_3 (x)

$CH_3-\overset{\overset{O}{\|}}{C}-CH_3$,

+

$CH_3-\overset{\overset{O}{\|}}{C}-OH$

+

$CH_3-\overset{\overset{O}{\|}}{C}-CH_2CH_2-\overset{\overset{O}{\|}}{C}-OH$

SECTION-B

٧١. ٠.٠١ mole of an organic compound (X) containing ١٠٪ hydrogen, on complete combustion produced ٠.٩ g H_2O. Molar mass of (X) is _____ $g\,mol^{-1}$.

Ans. (١٠٠)

Sol. Organic compound $\xrightarrow{\text{combustion}}$ H_2O
٠.٩gm

$\because$ mole of $H_2O = \frac{٠.٩}{١٨} = ٠.٠٥$ mole

$\therefore$ mole of H in $H_2O = ٠.٠٥ \times ٢ = ٠.١$ mole

= mole of H in ٠.٠١ mole Organic compound

$\therefore$ wt of H atom in ٠.٠١ mole compound $= ٠.١ \times ١$
$= ٠.١$ gm

$\therefore$ wt of H atom in one mole compound

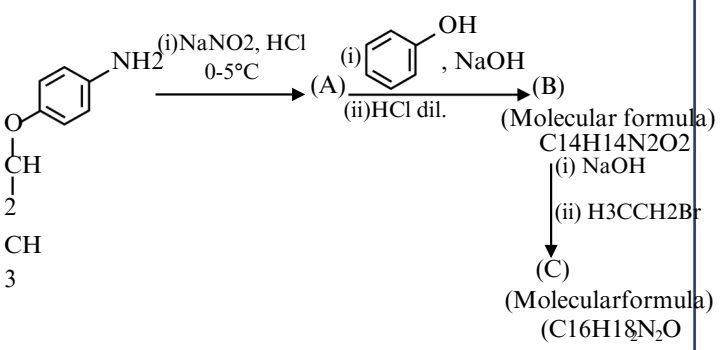
$= \frac{٠.١}{٠.٠١} \times ١٠ gm$

$\because$ wt. % of H $= \frac{\text{wt. of H in one mole compound} \times ١}{\text{Molar mass of compound}}$

$10 = \frac{١٠}{M} \times ١٠٠$

$\therefore$ $\boxed{M = ١٠}$

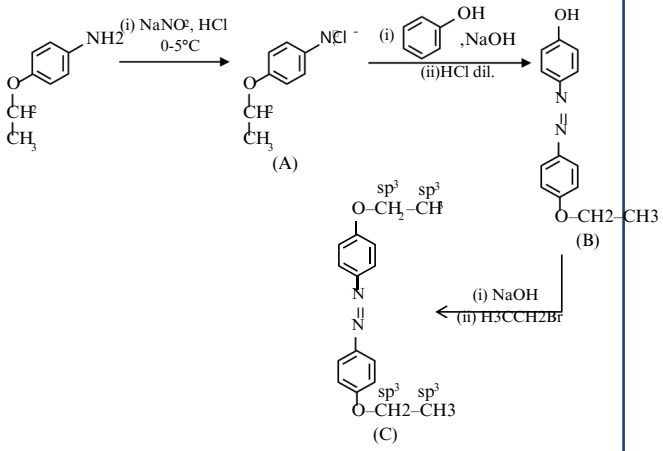
٧٢. Consider the following sequence of reactions.



Total number of sp hybridised carbon atoms in the major product C formed is _____.

Ans. (٤)

Sol.



٧٣. When ٨١.٠ g of aluminium is allowed to react with ١٢٨.٠ g of oxygen gas, the mass of aluminium oxide produced in grams is _____. (Nearest integer) Given : Molar mass of Al is ٢٧.٠ $g\,mol^{-1}$

Molar mass of O is ١٦.٠ $g\,mol^{-1}$

Ans. (١٥٣)

Sol. $4Al + 3O_2 \rightarrow 2Al_2O_3$

$\frac{٨١}{٢٧} = 3$ mole $\quad \frac{١٢٨}{٣٢} = 4$ mole

Limiting reagent

$\therefore$ mole of Al_2O_3 formed $= \frac{1}{2} \times 3$ mole

$\therefore$ wt. of Al_2O_3 formed $= \frac{3}{2} \times ١٠٢$

$= ١٥٣$ gm

٧٤. The bond dissociation enthalpy of X_2 H^o_{bond} calculated from the given data is _____ $kJ\,mol^{-1}$ (Nearest integer)

$MX(s) \rightarrow M(g) + X(g)$ $H^\circ_{lattice} = ٨٠٠\ kJ\,mol^{-1}$

$M(s) \rightarrow M(g)$ $H^\circ_{sub} = ١٠٠\ kJ\,mol^{-1}$

$M(g) \rightarrow M^+(g) + e^-(g)$ $H^\circ_i = ٥٠٠\ kJ\,mol^{-1}$

$X(g) + e^-(g) \rightarrow X^-(g)$ $H^\circ_{eg} = -٣٠٠\ kJ\,mol^{-1}$

$M(s) + \frac{1}{2}X_2(g) \rightarrow MX(s)$ $H^\circ_f = -٤٠٠\ kJ\,mol^{-1}$

Given : MX is a pure ionic compound and X forms a diatomic molecule X_2 is gaseous state

Ans. (٢٠٠)

Sol. $M_{(s)} + \frac{1}{2}X_{2(g)} \rightarrow MX_{(s)}$

$M_{(g)}$ $\quad$ $X_{(g)}$

$M^+_{(g)} + X^-_{(g)}$

$H_f(MX) = H_{sub}(M) + I.E.(M) + \frac{1}{2}B.E.(X-X) + EG(X) + L.E.(MX)$

$-٤٠٠ = (١٠٠) + (٥٠٠) + \frac{1}{2}(B.E.) + (-٣٠٠) + (-٨٠٠)$

$\therefore B.E. = ٢٠٠\ kJ\,mole^{-1}$